# Module 4: Market Efficiency (BUSFIN 4221 - Investments)

Andrei S. Gonçalves<sup>1</sup>

<sup>1</sup>Finance Department The Ohio State University

Fall 2016

# Outline

#### Overview

Efficient Market Hypothesis

EMH Tests: Return Predictability

EMH Tests: Active Investing

Behavioral Finance

t Hypothesis EMH Tests: Return Predictability EMH Tests: Active Investing Behavloral Finance

#### Module 1 - The Demand for Capital



vet Hypothesis - EMH Tests: Return Predictability - EMH Tests: Active Investing - Behavioral Finance

#### Module 1 - The Supply of Capital



Market Hypothesis EMH Tests: Return Predictability EMH Tests: Active Investing Behavioral Finance

#### Module 1 - Investment Principle

$$PV_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + dr_{t,h} \right)^{h}}$$

#### Module 2 - Portfolio Theory



#### Module 3 - Factor Models

$$\mathbb{E}[r_i] = r_f + \beta_i \cdot (\mathbb{E}[r_M] - r_f)$$
$$+ \beta_{i,A} \cdot \mathbb{E}[r_A - r_a]$$
$$+ \beta_{i,B} \cdot \mathbb{E}[r_B - r_b]$$
$$+ \dots$$

Market Hypothesis EMH Tests: Return Predictability EMH Tests: Active Investing Behavioral Finance

#### This Module: Market Efficiency

Prices incorporate all relevant information available up to time t



EMH Tests: EMH Tests: Return Predictability EMH Tests: Active Investing Behavioral Finance

#### This Module: Market Efficiency

Prices incorporate all relevant information available up to time t



et Hypothesis EMH Tests: Return Predictability EMH Tests: Active Investing Behavioral Finance

#### This Module: Market Efficiency

Prices incorporate all relevant information available up to time t

Prices incorporate all relevant information available up to time T



# Outline

Overview

Efficient Market Hypothesis

EMH Tests: Return Predictability

EMH Tests: Active Investing

Behavioral Finance

### This Section: Efficient Market Hypothesis (EMH)

Prices correctly incorporate all relevant information available up to time t

Prices correctly incorporate all relevant information available up to time T



- Example: After the recent presidential debate, the peso (Mexican currency) appreciated by 2% relative to the dollar
- What is the  $f(\cdot)$  function?
- What does it mean for f (·) to "correctly incorporate" all information?
- What information is relevant for prices?

- Example: After the recent presidential debate, the peso (Mexican currency) appreciated by 2% relative to the dollar
- What is the  $f(\cdot)$  function?
- What does it mean for f (·) to "correctly incorporate" all information?
- What information is relevant for prices?

- Example: After the recent presidential debate, the peso (Mexican currency) appreciated by 2% relative to the dollar
- What is the  $f(\cdot)$  function?
- What does it mean for f (·) to "correctly incorporate" all information?
- What information is relevant for prices?

 $P_t = f$  (Information available at time t)

- Example: After the recent presidential debate, the peso (Mexican currency) appreciated by 2% relative to the dollar
- What is the  $f(\cdot)$  function?
- What does it mean for *f*(·) to "correctly incorporate" all information?

• What information is relevant for prices?

- Example: After the recent presidential debate, the peso (Mexican currency) appreciated by 2% relative to the dollar
- What is the  $f(\cdot)$  function?
- What does it mean for  $f(\cdot)$  to "correctly incorporate" all information?
- What information is relevant for prices?

• In your "introduction to finance" class you used the definition of an interest rate to find:

$$PV = \frac{CF}{1+r} + \frac{CF}{(1+r)^2} + \frac{CF}{(1+r)^3} + \dots$$

$$\frac{|\log 20| s^{2}|_{1}}{(|s|, 2^{2}| -1)} = \frac{|\sin 20| s^{2}|_{1}}{(|s|, 2^{2}| -1)} = \frac{|s_{1}|^{2}}{(|s|, 2^{2}| -1)} = \frac{$$

• In your "introduction to finance" class you used the definition of an interest rate to find:

$$PV = \frac{CF}{1+r} + \frac{CF}{(1+r)^2} + \frac{CF}{(1+r)^3} + \dots$$

$$\frac{1}{2} \left[ \left( \log \frac{23}{2} \right) \left( \frac{3}{2} + \frac{1}{2} \left( \log \frac{23}{2} \right) \left( \frac{1}{2} + \frac{1}{2} \left( \log \frac{23}{2} + \frac{1}{2} \right) \right)^{\frac{1}{2}} + \frac{1}{2} \left( \log \frac{23}{2} + \frac{1}{2} + \frac{1}{2} \right)^{\frac{1}{2}} + \frac{1}{2} \left( \log \frac{23}{2} + \frac{1}{2} + \frac{1}{2} \right)^{\frac{1}{2}} = \frac{1}{2} \left( \cos \frac{23}{2} + \frac{1}{2} + \frac{1}{2} \right)^{\frac{1}{2}} = \frac{1}{2} \left( \cos \frac{23}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right)^{\frac{1}{2}} = \frac{1}{2} \left( \cos \frac{23}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right)^{\frac{1}{2}} = \frac{1}{2} \left( \cos \frac{23}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right)^{\frac{1}{2}} = \frac{1}{2} \left( \cos \frac{23}{2} + \frac{1}{2} +$$

• In your "introduction to finance" class you used the definition of an interest rate to find:

$$PV = rac{CF}{1+r} + rac{CF}{(1+r)^2} + rac{CF}{(1+r)^3} + \dots$$

$$P_{t} = \frac{\mathbb{E}_{t} \left[ CF_{t+1} \right]}{1 + \mathbb{E}_{t} \left[ r \right]} + \frac{\mathbb{E}_{t} \left[ CF_{t+2} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{2}} + \frac{\mathbb{E}_{t} \left[ CF_{t+3} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{3}} + \dots$$
$$= \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} = f \left( \text{Information at time } t \right)$$

• In your "introduction to finance" class you used the definition of an interest rate to find:

$$PV = rac{CF}{1+r} + rac{CF}{(1+r)^2} + rac{CF}{(1+r)^3} + \dots$$

$$P_{t} = \frac{\mathbb{E}_{t} \left[ CF_{t+1} \right]}{1 + \mathbb{E}_{t} \left[ r \right]} + \frac{\mathbb{E}_{t} \left[ CF_{t+2} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{2}} + \frac{\mathbb{E}_{t} \left[ CF_{t+3} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{3}} + \dots$$
$$= \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} = f \left( \text{Information at time } t \right)$$

• In your "introduction to finance" class you used the definition of an interest rate to find:

$$PV = rac{CF}{1+r} + rac{CF}{(1+r)^2} + rac{CF}{(1+r)^3} + \dots$$

$$P_{t} = \frac{\mathbb{E}_{t} \left[ CF_{t+1} \right]}{1 + \mathbb{E}_{t} \left[ r \right]} + \frac{\mathbb{E}_{t} \left[ CF_{t+2} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{2}} + \frac{\mathbb{E}_{t} \left[ CF_{t+3} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{3}} + \dots$$
$$= \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} = f (\text{Information at time } t)$$

• In your "introduction to finance" class you used the definition of an interest rate to find:

$$PV = rac{CF}{1+r} + rac{CF}{(1+r)^2} + rac{CF}{(1+r)^3} + \dots$$

$$P_{t} = \frac{\mathbb{E}_{t} \left[ CF_{t+1} \right]}{1 + \mathbb{E}_{t} \left[ r \right]} + \frac{\mathbb{E}_{t} \left[ CF_{t+2} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{2}} + \frac{\mathbb{E}_{t} \left[ CF_{t+3} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{3}} + \dots$$
$$= \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} = f \left( \text{Information at time } t \right)$$

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \longrightarrow \mathbb{E}_t [r] = dr$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t [CF_{t+h}]$
- Prices are too low:  $\widehat{CF}_{t+h} < \mathbb{E}_t \left[ CF_{t+h} \right]$
- Here is a Shark Tank episode showing how this principle works

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \times \qquad P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + dr_{t} \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \longrightarrow \mathbb{E}_t [r] = dr_t$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t [CF_{t+h}]$
- Prices are too low:  $\widehat{CF}_{t+h} < \mathbb{E}_t [CF_{t+h}]$
- Here is a Shark Tank episode showing how this principle works

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \times \qquad P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + dr_{t} \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] =$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t [CF_{t+h}]$
- Prices are too low:  $\widehat{\mathit{CF}}_{t+h} < \mathbb{E}_t \left[ \mathit{CF}_{t+h} 
  ight]$
- Here is a Shark Tank episode showing how this principle works

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \times \qquad P_{t} = \sum_{h=1}^{\infty} \frac{\widehat{CF}_{t+h}}{\left( 1 + dr_{t} \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r]$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t \left[ CF_{t+h} \right]$
- Prices are too low:  $\widehat{CF}_{t+h} < \mathbb{E}_t \left[ CF_{t+h} 
  ight]$
- Here is a Shark Tank episode showing how this principle works

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \times \qquad P_{t} = \sum_{h=1}^{\infty} \frac{\widehat{CF}_{t+h}}{\left( 1 + dr_{t} \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] = dr_t$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t [CF_{t+h}]$
- Prices are too low:  $\widehat{CF}_{t+h} < \mathbb{E}_t \left[ CF_{t+h} 
  ight]$
- Here is a Shark Tank episode showing how this principle works

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \times \qquad P_{t} = \sum_{h=1}^{\infty} \frac{\widehat{CF}_{t+h}}{\left( 1 + dr_{t} \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] = dr_t$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t \left[ CF_{t+h} \right]$
- Prices are too low:  $\widehat{CF}_{t+h} < \mathbb{E}_t \left[ CF_{t+h} 
  ight]$
- Here is a Shark Tank episode showing how this principle works

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \times \qquad P_{t} = \sum_{h=1}^{\infty} \frac{\widehat{CF}_{t+h}}{\left( 1 + dr_{t} \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] = dr_t$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t [CF_{t+h}]$
- Prices are too low:  $\widehat{CF}_{t+h} < \mathbb{E}_t [CF_{t+h}]$
- Here is a Shark Tank episode showing how this principle works

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \times \qquad P_{t} = \sum_{h=1}^{\infty} \frac{\widehat{CF}_{t+h}}{\left( 1 + dr_{t} \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] = dr_t$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t \left[ CF_{t+h} \right] \implies \mathbb{E}_t \left[ r \right] < dr_t$
- Prices are too low:  $\widehat{CF}_{t+h} < \mathbb{E}_t [CF_{t+h}]$
- Here is a Shark Tank episode showing how this principle works

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \times \qquad P_{t} = \sum_{h=1}^{\infty} \frac{\widehat{CF}_{t+h}}{\left( 1 + dr_{t} \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] = dr_t$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] < dr_t$
- Prices are too low:  $\widehat{CF}_{t+h} < \mathbb{E}_t [CF_{t+h}]$

• Here is a Shark Tank episode showing how this principle works

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \times \qquad P_{t} = \sum_{h=1}^{\infty} \frac{\widehat{CF}_{t+h}}{\left( 1 + dr_{t} \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] = dr_t$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t \left[ CF_{t+h} \right] \implies \mathbb{E}_t \left[ r \right] < dr_t$
- Prices are too low:  $\widehat{CF}_{t+h} < \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] > dr_t$

Here is a Shark Tank episode showing how this principle works

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \times \qquad P_{t} = \sum_{h=1}^{\infty} \frac{\widehat{CF}_{t+h}}{\left( 1 + dr_{t} \right)^{h}}$$

- *dr<sub>t</sub>* is the discount rate or "required rate of return". It is determined by investors aversion to the given security
- EMH Holds:  $\widehat{CF}_{t+h} = \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] = dr_t$
- Prices are too high:  $\widehat{CF}_{t+h} > \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] < dr_t$
- Prices are too low:  $\widehat{CF}_{t+h} < \mathbb{E}_t [CF_{t+h}] \implies \mathbb{E}_t [r] > dr_t$
- Here is a Shark Tank episode showing how this principle works

#### Correctly Incorporating Information: Summary

Prices correctly incorporate all relevant information available up to time t

Prices correctly incorporate all relevant information available up to time T



#### Correctly Incorporating Information: Summary










Efficient Market Hypothesis

Suppose investors forecast stock market dividend growth to be 6% per year. However, there is a new technology being introduced in the world and it will allow people to teleport from one place to another. This will make firms much more productive, which you know will induce a growth much higher than the 6% assumed by the market. What should you do?

- a) Nothing, EMH must hold in this scenario and, thus, the best action is to be passive
- b) You should <u>increase</u> your allocation to equities since they currently pay expected returns above market required rate of return
- c) You should <u>decrease</u> your allocation to equities since they currently pay expected returns <u>above</u> market required rate of return
- d) You should <u>increase</u> your allocation to equities since they currently pay expected returns <u>below</u> market required rate of return
- e) You should <u>decrease</u> your allocation to equities since they currently pay expected returns <u>below</u> market required rate of return

Efficient Market Hypothesis

Suppose investors forecast stock market dividend growth to be 6% per year. However, there is a new technology being introduced in the world and it will allow people to teleport from one place to another. This will make firms much more productive, which you know will induce a growth much higher than the 6% assumed by the market. What should you do?

- a) Nothing, EMH must hold in this scenario and, thus, the best action is to be passive
- b) You should <u>increase</u> your allocation to equities since they currently pay expected returns above market required rate of return
- c) You should <u>decrease</u> your allocation to equities since they currently pay expected returns <u>above</u> market required rate of return
- d) You should <u>increase</u> your allocation to equities since they currently pay expected returns <u>below</u> market required rate of return
- e) You should <u>decrease</u> your allocation to equities since they currently pay expected returns <u>below</u> market required rate of return

- If markets are efficient, then you can (correctly) expect to get the return you require when investing
- There is no way to "beat the market" consistently! If you find a security that is paying a great expected return, that is because markets dislike this security and, as such, require a high rate of return for holding it
- You should ask yourself what are the characteristics of a security that induce markets to dislike it (risk, illiquidity, ...??)
- Key implication for the practical world: you should not invest in active management
- Active managers cannot deliver returns above what you would require from them, but they charge high fees. Index funds provide a better alternative (low costs!)

- If markets are efficient, then you can (correctly) expect to get the return you require when investing
- There is no way to "beat the market" consistently! If you find a security that is paying a great expected return, that is because markets dislike this security and, as such, require a high rate of return for holding it
- You should ask yourself what are the characteristics of a security that induce markets to dislike it (risk, illiquidity, ...??)
- Key implication for the practical world: you should not invest in active management
- Active managers cannot deliver returns above what you would require from them, but they charge high fees. Index funds provide a better alternative (low costs!)

- If markets are efficient, then you can (correctly) expect to get the return you require when investing
- There is no way to "beat the market" consistently! If you find a security that is paying a great expected return, that is because markets dislike this security and, as such, require a high rate of return for holding it
- You should ask yourself what are the characteristics of a security that induce markets to dislike it (risk, illiquidity, ...??)
- Key implication for the practical world: you should not invest in active management
- Active managers cannot deliver returns above what you would require from them, but they charge high fees. Index funds provide a better alternative (low costs!)

- If markets are efficient, then you can (correctly) expect to get the return you require when investing
- There is no way to "beat the market" consistently! If you find a security that is paying a great expected return, that is because markets dislike this security and, as such, require a high rate of return for holding it
- You should ask yourself what are the characteristics of a security that induce markets to dislike it (risk, illiquidity, ...??)
- Key implication for the practical world: you should not invest in active management
- Active managers cannot deliver returns above what you would require from them, but they charge high fees. Index funds provide a better alternative (low costs!)

- If markets are efficient, then you can (correctly) expect to get the return you require when investing
- There is no way to "beat the market" consistently! If you find a security that is paying a great expected return, that is because markets dislike this security and, as such, require a high rate of return for holding it
- You should ask yourself what are the characteristics of a security that induce markets to dislike it (risk, illiquidity, ...??)
- Key implication for the practical world: you should not invest in active management
- Active managers cannot deliver returns above what you would require from them, but they charge high fees. Index funds provide a better alternative (low costs!)

- Market efficiency requires smart investors to incorporate information into prices all the time
- But if markets are efficient, there is no benefit in doing so (you cannot beat the market)
- As such, if you are smart you should be a passive investor
- But if smart investors become passive, there is nobody incorporating information into prices! This implies markets cannot be efficient!
- It must be the case that markets have at least some degree of inefficiency (at least to justify the existence of smart investors incorporating information into prices)

- Market efficiency requires smart investors to incorporate information into prices all the time
- But if markets are efficient, there is no benefit in doing so (you cannot beat the market)
- As such, if you are smart you should be a passive investor
- But if smart investors become passive, there is nobody incorporating information into prices! This implies markets cannot be efficient!
- It must be the case that markets have at least some degree of inefficiency (at least to justify the existence of smart investors incorporating information into prices)

- Market efficiency requires smart investors to incorporate information into prices all the time
- But if markets are efficient, there is no benefit in doing so (you cannot beat the market)
- As such, if you are smart you should be a passive investor
- But if smart investors become passive, there is nobody incorporating information into prices! This implies markets cannot be efficient!
- It must be the case that markets have at least some degree of inefficiency (at least to justify the existence of smart investors incorporating information into prices)

- Market efficiency requires smart investors to incorporate information into prices all the time
- But if markets are efficient, there is no benefit in doing so (you cannot beat the market)
- As such, if you are smart you should be a passive investor
- But if smart investors become passive, there is nobody incorporating information into prices! This implies markets cannot be efficient!
- It must be the case that markets have at least some degree of inefficiency (at least to justify the existence of smart investors incorporating information into prices)

- Market efficiency requires smart investors to incorporate information into prices all the time
- But if markets are efficient, there is no benefit in doing so (you cannot beat the market)
- As such, if you are smart you should be a passive investor
- But if smart investors become passive, there is nobody incorporating information into prices! This implies markets cannot be efficient!
- It must be the case that markets have at least some degree of inefficiency (at least to justify the existence of smart investors incorporating information into prices)

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

• Weak-form EMH:

relevant information = trading data

• Semistrong-form EMH:

relevant information = all publicly available information

• Strong-form EMH:

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

• Weak-form EMH:

#### relevant information = trading data

• Semistrong-form EMH:

relevant information = all publicly available information

• Strong-form EMH:

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

• Weak-form EMH:

relevant information = trading data

• Semistrong-form EMH:

relevant information = all publicly available information

• Strong-form EMH:

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

• Weak-form EMH:

relevant information = trading data

• Semistrong-form EMH:

relevant information = all publicly available information

• Strong-form EMH:

Efficient Market Hypothesis

#### Which of the following is a prediction of the Weak-form EMH?

- a) Analyzing the financial information of firms and investing accordingly cannot deliver expected returns above required rates of return
- b) If you work for Google and know that it will acquire Yahoo over the next year (which is a piece of information not released to the public yet), then there is no point in trading on this information (even if it was legal to do so)
- c) Observing prices going down over the last month should not give you any information on how to trade
- d) Cash flow estimates are identical to future realized cash flows, which rules out, for example, any possibility for surprises in earnings announcements
- e) Prices are always right in the sense that no matter which information one uses (of any source), he/she cannot reach a different price for any given security

Efficient Market Hypothesis

#### Which of the following is a prediction of the Weak-form EMH?

- a) Analyzing the financial information of firms and investing accordingly cannot deliver expected returns above required rates of return
- b) If you work for Google and know that it will acquire Yahoo over the next year (which is a piece of information not released to the public yet), then there is no point in trading on this information (even if it was legal to do so)
- **c)** Observing prices going down over the last month should not give you any information on how to trade
- d) Cash flow estimates are identical to future realized cash flows, which rules out, for example, any possibility for surprises in earnings announcements
- e) Prices are always right in the sense that no matter which information one uses (of any source), he/she cannot reach a different price for any given security

## Outline

Overview

Efficient Market Hypothesis

EMH Tests: Return Predictability

EMH Tests: Active Investing

**Behavioral Finance** 

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr$$

- If this is true, the EMH is equivalent to returns being unpredictable. Hence, prices follow a "Random Walk"
- EMH test  $\Rightarrow$  check whether returns are truly unpredictable
- Alternative versions of the EMH ⇒ No predictability by alternative sets of information

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E} \left[ r \right] = dr$$

- If this is true, the EMH is equivalent to returns being unpredictable. Hence, prices follow a "Random Walk"
- EMH test  $\Rightarrow$  check whether returns are truly unpredictable
- Alternative versions of the EMH ⇒ No predictability by alternative sets of information

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E} \left[ r \right] = dr$$

- <u>If this is true</u>, the EMH is equivalent to returns being unpredictable. Hence, prices follow a "Random Walk"
- EMH test  $\Rightarrow$  check whether returns are truly unpredictable
- Alternative versions of the EMH ⇒ No predictability by alternative sets of information

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E} \left[ r \right] = dr$$

- <u>If this is true</u>, the EMH is equivalent to returns being unpredictable. Hence, prices follow a "Random Walk"
- EMH test  $\Rightarrow$  check whether returns are truly unpredictable
- Alternative versions of the EMH ⇒ No predictability by alternative sets of information

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E} \left[ r \right] = dr$$

- <u>If this is true</u>, the EMH is equivalent to returns being unpredictable. Hence, prices follow a "Random Walk"
- EMH test  $\Rightarrow$  check whether returns are truly unpredictable
- Alternative versions of the EMH  $\Rightarrow$  No predictability by alternative sets of information

EMH Tests: Return Predictability

### Event Studies: Logic

Event that is known to affect  $E_t[CF_{t+h}]$ 

t

### Event Studies: Logic



Q: How long does it take for prices to incorporate the change?

## Event Studies: Logic



Q: How long does it take for prices to incorporate the change? EMH ➡ It incorporates instantaneously: <u>no predictability left</u>

## Event Studies: Logic

We can study returns after the event to see if there is any systematic pattern.



Q: How long does it take for prices to incorporate the change? EMH It incorporates instantaneously: <u>no predictability left</u>



Q: How long does it take for prices to incorporate the change? EMH ➡ It incorporates instantaneously: <u>no predictability left</u>

### Event Studies: Takeover Attempts\*



Source: Keown and Pinkerton (1981) - Merger Announcements and Insider Trading Activity

# Event Studies: CNBC (Mentioned on "Midday Call")\*




### Systematic Return Forecast: Motivation



Source: Vanguard Group (2012) - Forecasting Stock Returns: what Signals Matters and what do they say now?

- This graph shows (at each point in time) the equity market average return over the previous 10 years
- It is clear that there is substantial variation in average returns
- This motivates us to check whether long-run returns are predictable by variables known ahead of time

### Systematic Return Forecast: Motivation



Source: Vanguard Group (2012) - Forecasting Stock Returns: what Signals Matters and what do they say now?

- This graph shows (at each point in time) the equity market average return over the previous 10 years
- It is clear that there is substantial variation in average returns
- This motivates us to check whether long-run returns are predictable by variables known ahead of time

### Systematic Return Forecast: Motivation



Source: Vanguard Group (2012) - Forecasting Stock Returns: what Signals Matters and what do they say now?

- This graph shows (at each point in time) the equity market average return over the previous 10 years
- It is clear that there is substantial variation in average returns
- This motivates us to check whether long-run returns are predictable by variables known ahead of time

### Systematic Return Forecast: Logic



#### ctability

### Systematic Return Forecast: Logic





Systematic Return Forecast: Logic



### Systematic Return Forecast: Past Returns\*



Past 5 Year Excess Return

## Systematic Return Forecast: Price-to-Earnings $(P/E)^*$



Source: Malkiel (2003) - The Efficient Market Hypothesis and its Critics

### Which of the following is valid?

- a) All event studies presented provide evidence in support of the EMH
- **b)** There is substantial evidence of return predictability, which contradicts the EMH
- c) Returns do not vary over time and, thus, there is no point in looking for predictability in returns
- d) If the EMH is true, then prices must follow a "Random Walk" and returns must be unpredictable
- e) One way to interpret the evidence on return predictability is to think of time-varying required rates of return

Which of the following is valid?

- a) All event studies presented provide evidence in support of the EMH
- **b)** There is substantial evidence of return predictability, which contradicts the EMH
- c) Returns do not vary over time and, thus, there is no point in looking for predictability in returns
- d) If the EMH is true, then prices must follow a "Random Walk" and returns must be unpredictable
- e) One way to interpret the evidence on return predictability is to think of time-varying required rates of return

## Outline

EMH Tests: Active Investing

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

• Weak-form EMH:

 $\circ$  . Active investing based on trading data cannot generate lpha

• Semistrong-form EMH:

 $\sim$  Active investing based on public information cannot generate  $\alpha$ 

- Strong-form EMH:
  - Active investing does not generate or no matter what the information is used
  - This is rejected in the data (insider trading generates o).
    However, investing based on private information is illegal

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

• Weak-form EMH:

 $\circ~$  Active investing based on trading data cannot generate  $\alpha$ 

• Semistrong-form EMH:

Active investing based on public information cannot generate on

- Strong-form EMH:
  - Active investing does not generate to no matter which information is used
  - This is rejected in the data (insider trading generates a).
    However, investing based on private information is illegal

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

- Weak-form EMH:
  - $\circ~$  Active investing based on trading data cannot generate  $\alpha$
- Semistrong-form EMH:
  - Active investing based on public information cannot generate contraction.
- Strong-form EMH:
  - Active investing does not generate a no matter whatter information is used.
  - This is rejected in the data (insider trading generates a).
    However, investing based on private information is illegal

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

- Weak-form EMH:
  - $\circ~$  Active investing based on trading data cannot generate  $\alpha$
- <u>Semistrong-form</u> EMH:

 $\circ~$  Active investing based on public information cannot generate lpha

- Strong-form EMH:
  - Active investing does not generate a no matter what information is used.
  - This is rejected in the data (insider trading generates o).
    However, investing based on private information is illegal

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

- Weak-form EMH:
  - $\,\circ\,$  Active investing based on trading data cannot generate  $\alpha$
- Semistrong-form EMH:
  - $\,\circ\,$  Active investing based on public information cannot generate  $\alpha$
- Strong-form EMH:
  - Active investing does not generate or no matter what: information is used
  - This is rejected in the data (insider trading generates a).
    However, investing based on private information is illegal.

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

- Weak-form EMH:
  - $\circ~$  Active investing based on trading data cannot generate  $\alpha$
- Semistrong-form EMH:
  - $\circ~$  Active investing based on public information cannot generate  $\alpha$
- <u>Strong-form</u> EMH:
  - $\circ\,$  Active investing does not generate  $\alpha$  no matter what information is used
  - This is rejected in the data (insider trading generates α).
    However, investing based on private information is illegal

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

- Weak-form EMH:
  - $\,\circ\,$  Active investing based on trading data cannot generate  $\alpha$
- Semistrong-form EMH:
  - $\,\circ\,$  Active investing based on public information cannot generate  $\alpha$
- <u>Strong-form</u> EMH:
  - $\circ\,$  Active investing does not generate  $\alpha$  no matter what information is used
  - This is rejected in the data (insider trading generates α).
    However, investing based on private information is illegal

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}} \qquad \text{and} \qquad \mathbb{E}_{t} \left[ r \right] = dr_{t}$$

- Weak-form EMH:
  - $\circ~$  Active investing based on trading data cannot generate  $\alpha$
- Semistrong-form EMH:
  - $\,\circ\,$  Active investing based on public information cannot generate  $\alpha$
- <u>Strong-form</u> EMH:
  - $\circ\,$  Active investing does not generate  $\alpha$  no matter what information is used
  - This is rejected in the data (insider trading generates α).
    However, investing based on private information is illegal

### Mutual Funds: Returns\*



Source: Bodie, Kane and Marcus (10th ed) - Investments

#### EMH Tests: Active Investing

### Mutual Funds: $\alpha^*$



Source: Bodie, Kane and Marcus (10th ed) - Investments

### Mutual Funds: $\alpha$ Persistence\*



Source: Bollen and Busse (2004) - Short-Term Persistence in Mutual Fund Performance





 $\hat{x}$ : any characteristic of interest



 $\hat{x}$ : any characteristic of interest



 $\hat{x}$ : any characteristic of interest



5



## Dynamic Strategies ("Anomalies"): Size Effect\*



Ken French's data library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\_library.html

## Dynamic Strategies ("Anomalies"): Value Effect\*



Ken French's data library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\_library.html

# Dynamic Strategies ("Anomalies"): Momentum\*



Ken French's data library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\_library.html

### Which of the following is valid?

- a) If the Semistrong EMH is true, then any two strategies based on public information should have identical expected return
- b) All versions of the EMH imply that nobody can generate  $\alpha$  in financial markets no matter what information they use
- c) Insider trading does not generate  $\alpha$ , which is evidence in favor of the Strong-form EMH
- d) Most actively managed mutual funds do not consistently generate  $\alpha$  and this goes in favor of the EMH
- e) There is substantial variation in expected returns across stocks and part of this variation can be captured with dynamic trading strategies. This is strong evidence against the EMH

### Which of the following is valid?

- a) If the Semistrong EMH is true, then any two strategies based on public information should have identical expected return
- b) All versions of the EMH imply that nobody can generate  $\alpha$  in financial markets no matter what information they use
- c) Insider trading does not generate  $\alpha$ , which is evidence in favor of the Strong-form EMH
- d) Most actively managed mutual funds do not consistently generate  $\alpha$  and this goes in favor of the EMH
- e) There is substantial variation in expected returns across stocks and part of this variation can be captured with dynamic trading strategies. This is strong evidence against the EMH

## Outline

Overview

Efficient Market Hypothesis

EMH Tests: Return Predictability

EMH Tests: Active Investing

**Behavioral Finance** 

## Systematic Mistakes + Limits to Arbitrage $\Rightarrow$ EMH fails

- EMH hypothesis says that investors compete until  $\mathbb{E}_t[r] = dr_t$
- The EMH fails whenever  $\mathbb{E}_t[r] \neq dr_t$
- For this to happen, investors need to make similar mistakes (if each investor makes a different mistake, they average to zero and prices are not affected)
- Moreover, smart investors must be unable to profit from the mistakes of other investors without taking a lot of risk (otherwise they would adjust prices)
- Behavioral Finance argues that both aspects are true in reality:
  Systematic Mistakes + Limits to Arbitrage ⇒ EMH fails
- EMH hypothesis says that investors compete until  $\mathbb{E}_t[r] = dr_t$
- The EMH fails whenever  $\mathbb{E}_t[r] \neq dr_t$
- For this to happen, investors need to make similar mistakes (if each investor makes a different mistake, they average to zero and prices are not affected)
- Moreover, smart investors must be unable to profit from the mistakes of other investors without taking a lot of risk (otherwise they would adjust prices)
- Behavioral Finance argues that both aspects are true in reality:
  Systematic Mistakes + Limits to Arbitrage ⇒ EMH fails

- EMH hypothesis says that investors compete until  $\mathbb{E}_t[r] = dr_t$
- The EMH fails whenever  $\mathbb{E}_t[r] \neq dr_t$
- For this to happen, investors need to make similar mistakes (if each investor makes a different mistake, they average to zero and prices are not affected)
- Moreover, smart investors must be unable to profit from the mistakes of other investors without taking a lot of risk (otherwise they would adjust prices)
- Behavioral Finance argues that both aspects are true in reality:
  Systematic Mistakes + Limits to Arbitrage ⇒ EMH fails

- EMH hypothesis says that investors compete until  $\mathbb{E}_t[r] = dr_t$
- The EMH fails whenever  $\mathbb{E}_t[r] \neq dr_t$
- For this to happen, investors need to make similar mistakes (if each investor makes a different mistake, they average to zero and prices are not affected)
- Moreover, smart investors must be unable to profit from the mistakes of other investors without taking a lot of risk (otherwise they would adjust prices)
- Behavioral Finance argues that both aspects are true in reality:
  Systematic Mistakes + Limits to Arbitrage ⇒ EMH fails

- EMH hypothesis says that investors compete until  $\mathbb{E}_t[r] = dr_t$
- The EMH fails whenever  $\mathbb{E}_t[r] \neq dr_t$
- For this to happen, investors need to make similar mistakes (if each investor makes a different mistake, they average to zero and prices are not affected)
- Moreover, smart investors must be unable to profit from the mistakes of other investors without taking a lot of risk (otherwise they would adjust prices)
- Behavioral Finance argues that both aspects are true in reality:
  Systematic Mistakes + Limits to Arbitrage ⇒ EMH fails

## Evidence: Abnormal Return one Day after Soccer Match\*



Source: Edmans, García and Norli (2007) - Sports Sentiment and Stock Returns

### Evidence: Adding ".com" on Company Name\*



#### Figure 1. Cumulative abnormal returns earned around the announcement date by firms changing their names to dotcom names.

Source: Cooper, Dimitrov and Rau (2001) - A Rose.com by any other Name

Evidence: "CUBA" Fund\*



Source: Thaler (2016) - Behavioral Economics: Past, Present, and Future

## Systematic Mistakes: Mental Accounting

- Did your parents create a "College Fund" for you?
- Mental Accounting bias:

"Tendency for people to separate their money into separate accounts based on a variety of subjective criteria, like the source of the money and intent for each account"

- Application to Investments:
  - Investors are more likely to sell stocks with gains than the ones with loss ("disposition effect")
  - This goes against the tax minimization strategy

### Systematic Mistakes: Mental Accounting

- Did your parents create a "College Fund" for you?
- Mental Accounting bias:

"Tendency for people to separate their money into separate accounts based on a variety of subjective criteria, like the source of the money and intent for each account"

- Application to Investments:
  - Investors are more likely to sell stocks with gains than the ones with loss ("disposition effect")
  - This goes against the tax minimization strategy

### Systematic Mistakes: Mental Accounting

- Did your parents create a "College Fund" for you?
- Mental Accounting bias:

"Tendency for people to separate their money into separate accounts based on a variety of subjective criteria, like the source of the money and intent for each account"

- Application to Investments:
  - Investors are more likely to sell stocks with gains than the ones with loss ("disposition effect")
  - This goes against the tax minimization strategy

## Systematic Mistakes: Availability

• Which is more likely:

being killed by (i) a Shark or (ii) a falling vending machine?

The availability bias:

"Probability is estimated by the ease with which similar instances or associations can be brought to mind"

- Application to Investments:
  - Malmendier and Nagel (2001) find that individuals experiences affect risk taking
  - Individuals are less willing to participate in stock market if they lived through the Great Depression

## Systematic Mistakes: Availability

• Which is more likely:

being killed by (i) a Shark or (ii) a falling vending machine?

• The availability bias:

"Probability is estimated by the ease with which similar instances or associations can be brought to mind"

- Application to Investments:
  - Malmendier and Nagel (2001) find that individuals experiences affect risk taking
  - Individuals are less willing to participate in stock market if they lived through the Great Depression

## Systematic Mistakes: Availability

• Which is more likely:

being killed by (i) a Shark or (ii) a falling vending machine?

• The availability bias:

"Probability is estimated by the ease with which similar instances or associations can be brought to mind"

- Application to Investments:
  - Malmendier and Nagel (2001) find that individuals experiences affect risk taking
  - Individuals are less willing to participate in stock market if they lived through the Great Depression

## Systematic Mistakes: Representativeness

• Choose 6 numbers from 1 to 60 to play in the lottery:

(i) 1, 2, 3, 4, 5, 6 or (ii) 20, 32, 7, 43, 59, 12?

• The representativeness bias:

"The probability that event X belongs to set Y is judged on the basis of how similar X is to the stereotype of Y"

- Application to Investments:
  - If a given firm has characteristics similar to typical risky firms, then investors might believe the firm is very risky regardless of whether it really is. This can lead to  $\mathbb{E}_t[\mathbf{r}]$  being related to firm characteristics as oppose to firm risk

### Systematic Mistakes: Representativeness

• Choose 6 numbers from 1 to 60 to play in the lottery:

(i) 1, 2, 3, 4, 5, 6 or (ii) 20, 32, 7, 43, 59, 12?

• The representativeness bias:

"The probability that event X belongs to set Y is judged on the basis of how similar X is to the stereotype of Y"

- Application to Investments:
  - If a given firm has characteristics similar to typical risky firms, then investors might believe the firm is very risky regardless of whether it really is. This can lead to  $\mathbb{E}_t[r]$  being related to firm characteristics as oppose to firm risk

## Systematic Mistakes: Representativeness

• Choose 6 numbers from 1 to 60 to play in the lottery:

(i) 1, 2, 3, 4, 5, 6 or (ii) 20, 32, 7, 43, 59, 12?

• The representativeness bias:

"The probability that event X belongs to set Y is judged on the basis of how similar X is to the stereotype of Y"

- Application to Investments:
  - If a given firm has characteristics similar to typical risky firms, then investors might believe the firm is very risky regardless of whether it really is. This can lead to  $\mathbb{E}_t[r]$  being related to firm characteristics as oppose to firm risk

# Systematic Mistakes: Local Representativeness

#### • "Hot hand" fallacy in Basketball:

• The local representativeness bias:

"Exaggerate how likely it is that a small sample resembles the parent population from which it is drawn"

- Application to Investments:
  - Money chasing Mutual Funds past performance
  - Potential explanation for Momentum

## Systematic Mistakes: Local Representativeness

- "Hot hand" fallacy in Basketball:
- The local representativeness bias:

"Exaggerate how likely it is that a small sample resembles the parent population from which it is drawn"

- Application to Investments:
  - Money chasing Mutual Funds past performance
  - Potential explanation for Momentum

## Systematic Mistakes: Local Representativeness

- "Hot hand" fallacy in Basketball:
- The local representativeness bias:

"Exaggerate how likely it is that a small sample resembles the parent population from which it is drawn"

- Application to Investments:
  - Money chasing Mutual Funds past performance
  - Potential explanation for Momentum

- Is your driving ability above median?
  - 50% should be below and 50% above
  - $\circ~93\%$  of U.S. students estimate they are above median
- The overconfidence bias:
  - "People overestimate the precision of their beliefs or forecasts. They are 'surprised' too often"
- Application to Investments:
  - Men trade much more often than women (45% more) and have much lower net returns as a result (0.93% lower per year)

- Is your driving ability above median?
  - $\circ~50\%$  should be below and 50% above
  - $\circ$  93% of U.S. students estimate they are above median
- The overconfidence bias:

- Application to Investments:
  - Men trade much more often than women (45% more) and have much lower net returns as a result (0.93% lower per year)

- Is your driving ability above median?
  - $\circ~50\%$  should be below and 50% above
  - $\circ~93\%$  of U.S. students estimate they are above median
- The overconfidence bias:

- Application to Investments:
  - Men trade much more often than women (45% more) and have much lower net returns as a result (0.93% lower per year)

- Is your driving ability above median?
  - $\circ~50\%$  should be below and 50% above
  - $\circ~93\%$  of U.S. students estimate they are above median
- The overconfidence bias:

- Application to Investments:
  - Men trade much more often than women (45% more) and have much lower net returns as a result (0.93% lower per year)

- Is your driving ability above median?
  - $\circ~50\%$  should be below and 50% above
  - $\circ~93\%$  of U.S. students estimate they are above median
- The overconfidence bias:

- Application to Investments:
  - Men trade much more often than women (45% more) and have much lower net returns as a result (0.93% lower per year)

# Systematic Mistakes: Trading Activity x Returns\*



Source: Barber and Odean (2001) - Boys will be boys: gender, overconfidence, and common stock investment

- Grinblatt et al (2011): High IQ investors are not as biased
- If "arbitrageurs" can easily take advantage of other investor's mistakes, then cognitive biases cannot have an effect on prices
- They can create very profitable strategies taking little risk: as they do it, prices adjust
- Three aspects of financial markets limit their actions:
  - Fundamental Risk (trading horizon matters!).
  - Implementation Costs/Restrictions
  - Model Risk (is this really a mispriced security?)

- Grinblatt et al (2011): High IQ investors are not as biased
- If "arbitrageurs" can easily take advantage of other investor's mistakes, then cognitive biases cannot have an effect on prices
- They can create very profitable strategies taking little risk: as they do it, prices adjust
- Three aspects of financial markets limit their actions:
  - Fundamental Risk (trading horizon matters!).
  - Implementation Costs/Restrictions
  - Model Risk (is this really a mispriced security?)

- Grinblatt et al (2011): High IQ investors are not as biased
- If "arbitrageurs" can easily take advantage of other investor's mistakes, then cognitive biases cannot have an effect on prices
- They can create very profitable strategies taking little risk: as they do it, prices adjust
- Three aspects of financial markets limit their actions:
  - Fundamental Risk (trading horizon mattersl)
  - Implementation Costs/Restrictions
  - Model Risk (is this really a mispriced security?)

- Grinblatt et al (2011): High IQ investors are not as biased
- If "arbitrageurs" can easily take advantage of other investor's mistakes, then cognitive biases cannot have an effect on prices
- They can create very profitable strategies taking little risk: as they do it, prices adjust
- Three aspects of financial markets limit their actions:
  - Fundamental Risk (trading horizon matters!)
  - Implementation Costs/Restrictions
  - Model Risk (is this really a mispriced security?)

- Grinblatt et al (2011): High IQ investors are not as biased
- If "arbitrageurs" can easily take advantage of other investor's mistakes, then cognitive biases cannot have an effect on prices
- They can create very profitable strategies taking little risk: as they do it, prices adjust
- Three aspects of financial markets limit their actions:
  - Fundamental Risk (trading horizon matters!)
  - Implementation Costs/Restrictions
  - Model Risk (is this really a mispriced security?)

- Grinblatt et al (2011): High IQ investors are not as biased
- If "arbitrageurs" can easily take advantage of other investor's mistakes, then cognitive biases cannot have an effect on prices
- They can create very profitable strategies taking little risk: as they do it, prices adjust
- Three aspects of financial markets limit their actions:
  - Fundamental Risk (trading horizon matters!)
  - Implementation Costs/Restrictions
  - Model Risk (is this really a mispriced security?)

- Grinblatt et al (2011): High IQ investors are not as biased
- If "arbitrageurs" can easily take advantage of other investor's mistakes, then cognitive biases cannot have an effect on prices
- They can create very profitable strategies taking little risk: as they do it, prices adjust
- Three aspects of financial markets limit their actions:
  - Fundamental Risk (trading horizon matters!)
  - Implementation Costs/Restrictions
  - Model Risk (is this really a mispriced security?)

- EMH is clearly not a complete description of reality. The question is: how good is it?
- Proponents of behavioral finance argue that EMH completely ignores the way people act in reality and, thus, fails miserably
- EMH supporters indicate that most investors are better off if they act "as if" EMH were true (even though EMH is not literally true). Moreover, the "clear evidence" for mispricing is not systematic (they are more like anecdotes)
- Finally, they argue that behavioral finance does not provide a reasonable alternative way to think about investments since it is "unstructured" (each bias is used to "explain" a different piece of evidence and they often contradict each other)

- EMH is clearly not a complete description of reality. The question is: how good is it?
- Proponents of behavioral finance argue that EMH completely ignores the way people act in reality and, thus, fails miserably
- EMH supporters indicate that most investors are better off if they act "as if" EMH were true (even though EMH is not literally true). Moreover, the "clear evidence" for mispricing is not systematic (they are more like anecdotes)
- Finally, they argue that behavioral finance does not provide a reasonable alternative way to think about investments since it is "unstructured" (each bias is used to "explain" a different piece of evidence and they often contradict each other)

- EMH is clearly not a complete description of reality. The question is: how good is it?
- Proponents of behavioral finance argue that EMH completely ignores the way people act in reality and, thus, fails miserably
- EMH supporters indicate that most investors are better off if they act "as if" EMH were true (even though EMH is not literally true). Moreover, the "clear evidence" for mispricing is not systematic (they are more like anecdotes)
- Finally, they argue that behavioral finance does not provide a reasonable alternative way to think about investments since it is "unstructured" (each bias is used to "explain" a different piece of evidence and they often contradict each other)

- EMH is clearly not a complete description of reality. The question is: how good is it?
- Proponents of behavioral finance argue that EMH completely ignores the way people act in reality and, thus, fails miserably
- EMH supporters indicate that most investors are better off if they act "as if" EMH were true (even though EMH is not literally true). Moreover, the "clear evidence" for mispricing is not systematic (they are more like anecdotes)
- Finally, they argue that behavioral finance does not provide a reasonable alternative way to think about investments since it is "unstructured" (each bias is used to "explain" a different piece of evidence and they often contradict each other)
Regarding systematic biases inherent in human psychology and their influence in investment decisions:

- a) Mental accounting can induce investors to have biased probability estimates
- **b)** Overconfidence can induce prices to be above fundamental value, but not below fundamental value
- c) Availability can make investors to behave as if they are too risk averse even when there is no plausible justification for it. However, it cannot make investors to behave as less risk averse than they really are
- d) Local representativeness can lead investors to believe that prices will continue to rise if they have been rising in the recent past (it is a potential explanation for the Momentum effect)
- e) Representativeness can make investors to rely too much on new information and to disregard important information previously known

Regarding systematic biases inherent in human psychology and their influence in investment decisions:

- a) Mental accounting can induce investors to have biased probability estimates
- b) Overconfidence can induce prices to be above fundamental value, but not below fundamental value
- c) Availability can make investors to behave as if they are too risk averse even when there is no plausible justification for it. However, it cannot make investors to behave as less risk averse than they really are
- d) Local representativeness can lead investors to believe that prices will continue to rise if they have been rising in the recent past (it is a potential explanation for the Momentum effect)
- e) Representativeness can make investors to rely too much on new information and to disregard important information previously known

$$1 + \mathbb{E}_t \left[ r_{t+1} \right] = \mathbb{E}_t \left[ \frac{P_{t+1} + CF_{t+1}}{P_t} \right]$$

$$\begin{split} & \stackrel{\circ}{\mathsf{P}}_{t} = \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{P_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \right] \\ & = \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{1}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \cdot \left( \frac{CF_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} + \frac{P_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} \right) \right] \\ & = \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} \right)} + \frac{P_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} \right) \right] \\ & \vdots \\ & = \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} \right)} + \dots \right] + \mathbb{E}_{t} \left[ \frac{P_{t+\infty}}{\prod_{k=1}^{\infty} \left( 1 + \mathbb{E}_{k} \left[ r_{k+1} \right] \right)} \right] \\ & = \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} + \dots \right] \right] \\ & P_{t} = \sum_{h=1}^{\infty} \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right] + \left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right)} \right] \end{aligned}$$

$$\begin{split} + \mathbb{E}_{t} \left[ r_{t+1} \right] &= \mathbb{E}_{t} \left[ \frac{P_{t+1} + CF_{t+1}}{P_{t}} \right] \\ &\downarrow \\ P_{t} &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{P_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{1}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \cdot \left( \frac{CF_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} + \frac{P_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} \right) \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} + \frac{P_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} + \dots \right] + \mathbb{E}_{t} \left[ \frac{P_{t+\infty}}{\prod_{k=1}^{\infty} \left( 1 + \mathbb{E}_{k} \left[ r_{k+1} \right] \right)} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \dots \right] + \mathbb{E}_{t} \left[ \frac{P_{t+\infty}}{\prod_{k=1}^{\infty} \left( 1 + \mathbb{E}_{k} \left[ r_{k+1} \right] \right)} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \dots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right)} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \dots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right)} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \dots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \left[ r_{t+2} \right] \right]} + \dots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+2}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \left[ r_{t+2} \right] \left[ r_{t+2} \right] \left[ r_{t+2} \right]} + \dots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+2}}{1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \left[ r_{t+2} \right] \left[ r_{t+2} \left[ r_{t+2} \right] \left[ r_{t+2} \right] \left[ r_{t+2} \left[ r_{t+2} \right] \left[ r_{t+2} \right] \left[ r_{t+2} \left[ r_{t+2} \right] \left[ r_{t+2} \left[ r_{t+2} \right] \left[ r_$$

$$\begin{aligned} + \mathbb{E}_{t} \left[ r_{t+1} \right] &= \mathbb{E}_{t} \left[ \frac{P_{t+1} + CF_{t+1}}{P_{t}} \right] \\ &\downarrow \\ P_{t} &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{P_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{1}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \cdot \left( \frac{CF_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} + \frac{P_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} \right) \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+1} \right]) \cdot (1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right])} + \frac{P_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+1} \right]) \cdot (1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right])} \right] \\ &: \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+1} \right]) \cdot (1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right])} + \ldots \right] + \mathbb{E}_{t} \left[ \frac{P_{t+\infty}}{\prod_{k=t}^{\infty} (1 + \mathbb{E}_{k} \left[ r_{k+1} \right])} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+1} \right]) \cdot (1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right])} + \ldots \right] \\ &P_{t} = \sum_{k=1}^{\infty} \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+1} \right]) \cdot (1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right])} + \ldots \right] \end{aligned}$$

$$\begin{aligned} 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] &= \mathbb{E}_{t} \left[ \frac{P_{t+1} + CF_{t+1}}{P_{t}} \right] \\ &\downarrow \\ P_{t} &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{P_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{1}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \cdot \left( \frac{CF_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} + \frac{P_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} \right) \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} + \frac{P_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} + \ldots \right] + \mathbb{E}_{t} \left[ \frac{P_{t+\infty}}{\prod_{k=1}^{\infty} \left( 1 + \mathbb{E}_{k} \left[ r_{k+1} \right] \right)} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \ldots \right] + \mathbb{E}_{t} \left[ \frac{P_{t+\infty}}{\prod_{k=1}^{\infty} \left( 1 + \mathbb{E}_{k} \left[ r_{k+1} \right] \right)} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \ldots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \ldots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \ldots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \ldots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \ldots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \ldots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+2} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \ldots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+2} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \ldots \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+2}}{1 + \mathbb{E}_{t} \left[ r_{t+2} \right]} + \frac{CF_{t+2}}{1 + \mathbb{E}_{t} \left[ r_{t+2} \right]} + \frac{CF_{t+2}}{1 + \mathbb{E}_{t} \left[ r_{t+$$

$$\begin{aligned} + \mathbb{E}_{t} \left[ r_{t+1} \right] &= \mathbb{E}_{t} \left[ \frac{P_{t+1} + CF_{t+1}}{P_{t}} \right] \\ &\downarrow \\ P_{t} &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{P_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{1}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \cdot \left( \frac{CF_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} + \frac{P_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} \right) \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+1} \right]) \cdot (1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right])} + \frac{P_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+1} \right]) \cdot (1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right])} \right] \\ &\vdots \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+1} \right]) \cdot (1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right])} + \ldots \right] + \mathbb{E}_{t} \left[ \frac{P_{t+\infty}}{\prod_{k=t}^{\infty} (1 + \mathbb{E}_{k} \left[ r_{k+1} \right])} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+2} \right])} + \ldots \right] + \mathbb{E}_{t} \left[ \frac{P_{t+\infty}}{\prod_{k=t}^{\infty} (1 + \mathbb{E}_{k} \left[ r_{k+1} \right])} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+2} \right])} + \ldots \right] \\ &\rho_{t} = \sum_{k=0}^{\infty} \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+1} \right])} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+2} \right])} + \ldots \right] \\ &\rho_{t} = \sum_{k=0}^{\infty} \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+2} \right])} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+2} \right])} + \ldots \right] \\ &\rho_{t} = \sum_{k=0}^{\infty} \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+2} \right])} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+2} \right])} + \ldots \right] \\ &\rho_{t} = \sum_{k=0}^{\infty} \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t} \left[ r_{t+2} \right])} + \frac{CF_{t+2}}{(1 +$$

$$\begin{aligned} + \mathbb{E}_{t} \left[ r_{t+1} \right] &= \mathbb{E}_{t} \left[ \frac{P_{t+1} + CF_{t+1}}{P_{t}} \right] \\ &\downarrow \\ P_{t} &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{P_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{1}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} \cdot \left( \frac{CF_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} + \frac{P_{t+2}}{1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right]} \right) \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} + \frac{P_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} \right] \\ &\vdots \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) + \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} + \dots \right] + \mathbb{E}_{t} \left[ \frac{P_{t+\infty}}{\prod_{k=t}^{\infty} \left( 1 + \mathbb{E}_{k} \left[ r_{k+1} \right] \right)} \right] \\ &= \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+2} \right] \right)} + \dots \right] \right] \\ &P_{t} = \sum_{m=1}^{\infty} \mathbb{E}_{t} \left[ \frac{CF_{t+1}}{1 + \mathbb{E}_{t} \left[ r_{t+1} \right]} + \frac{CF_{t+2}}{\left( 1 + \mathbb{E}_{t} \left[ r_{t+1} \right] \right) \cdot \left( 1 + \mathbb{E}_{t+1} \left[ r_{t+2} \right] \right)} + \dots \right] \end{aligned}$$

$$\begin{split} + \mathbb{E}_{t}\left[r_{t+1}\right] &= \mathbb{E}_{t}\left[\frac{P_{t+1} + CF_{t+1}}{P_{t}}\right] \\ &\downarrow \\ P_{t} &= \mathbb{E}_{t}\left[\frac{CF_{t+1}}{1 + \mathbb{E}_{t}\left[r_{t+1}\right]} + \frac{P_{t+1}}{1 + \mathbb{E}_{t}\left[r_{t+1}\right]}\right] \\ &= \mathbb{E}_{t}\left[\frac{CF_{t+1}}{1 + \mathbb{E}_{t}\left[r_{t+1}\right]} + \frac{1}{1 + \mathbb{E}_{t}\left[r_{t+1}\right]} \cdot \left(\frac{CF_{t+2}}{1 + \mathbb{E}_{t+1}\left[r_{t+2}\right]} + \frac{P_{t+2}}{1 + \mathbb{E}_{t+1}\left[r_{t+2}\right]}\right)\right] \\ &= \mathbb{E}_{t}\left[\frac{CF_{t+1}}{1 + \mathbb{E}_{t}\left[r_{t+1}\right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t}\left[r_{t+1}\right]) \cdot (1 + \mathbb{E}_{t+1}\left[r_{t+2}\right])} + \frac{P_{t+2}}{(1 + \mathbb{E}_{t}\left[r_{t+1}\right]) \cdot (1 + \mathbb{E}_{t+1}\left[r_{t+2}\right])}\right] \\ &\vdots \\ &= \mathbb{E}_{t}\left[\frac{CF_{t+1}}{1 + \mathbb{E}_{t}\left[r_{t+1}\right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t}\left[r_{t+1}\right]) \cdot (1 + \mathbb{E}_{t+1}\left[r_{t+2}\right])} + ...\right] + \mathbb{E}_{t}\left[\frac{P_{t+\infty}}{\prod_{k=t}^{\infty} (1 + \mathbb{E}_{k}\left[r_{k+1}\right])}\right] \\ &= \mathbb{E}_{t}\left[\frac{CF_{t+1}}{1 + \mathbb{E}_{t}\left[r_{t+1}\right]} + \frac{CF_{t+2}}{(1 + \mathbb{E}_{t}\left[r_{t+1}\right] \cdot (1 + \mathbb{E}_{t+1}\left[r_{t+2}\right])} + ...\right] \right] \end{split}$$

$$P_{t} = \sum_{h=1}^{\infty} \mathbb{E}_{t} \left[ \frac{CF_{t+h}}{\prod\limits_{k=t}^{t+h-1} (1 + \mathbb{E}_{k} [r_{k+1}])} \right]$$

If we assume that (i) E<sub>t</sub> [r<sub>t+1</sub>] = E<sub>t</sub> [r<sub>t+2</sub>] ≡ E<sub>t</sub> [r] and (ii) shocks to cash flows are independent of shocks to expected returns, then we have:

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}}$$

$$P_{t} = \sum_{h=1}^{\infty} \mathbb{E}_{t} \left[ \frac{CF_{t+h}}{\prod\limits_{k=t}^{t+h-1} (1 + \mathbb{E}_{k} [r_{k+1}])} \right]$$

If we assume that (i) E<sub>t</sub> [r<sub>t+1</sub>] = E<sub>t</sub> [r<sub>t+2</sub>] ≡ E<sub>t</sub> [r] and (ii) shocks to cash flows are independent of shocks to expected returns, then we have:

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}}$$

$$P_{t} = \sum_{h=1}^{\infty} \mathbb{E}_{t} \left[ \frac{CF_{t+h}}{\prod\limits_{k=t}^{t+h-1} (1 + \mathbb{E}_{k} [r_{k+1}])} \right]$$

If we assume that (i) E<sub>t</sub> [r<sub>t+1</sub>] = E<sub>t</sub> [r<sub>t+2</sub>] ≡ E<sub>t</sub> [r] and (ii) shocks to cash flows are independent of shocks to expected returns, then we have:

$$P_{t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_{t} \left[ CF_{t+h} \right]}{\left( 1 + \mathbb{E}_{t} \left[ r \right] \right)^{h}}$$