Module 3: Factor Models

(BUSFIN 4221 - Investments)

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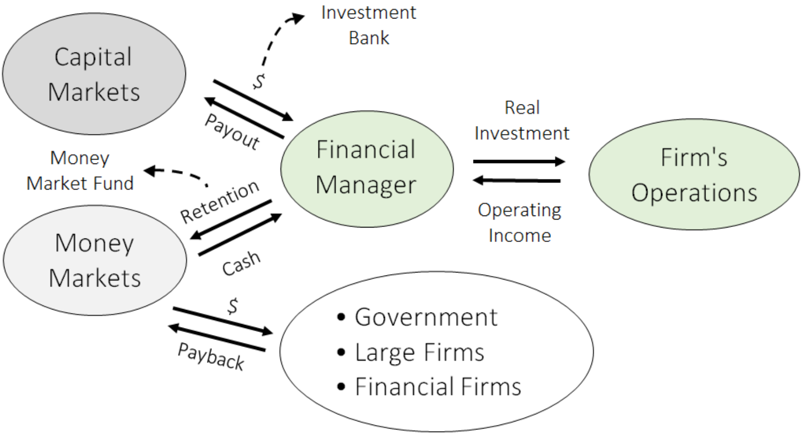
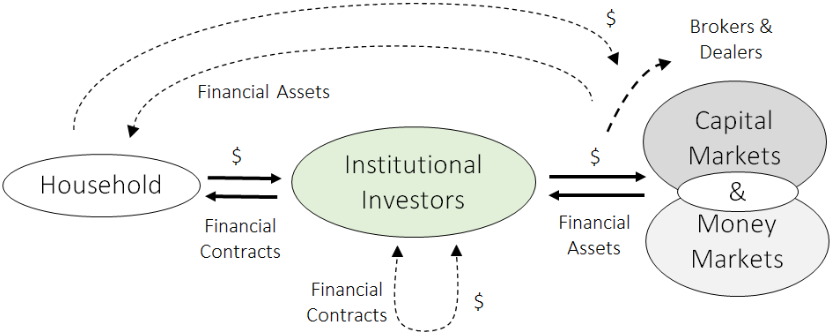
Module 1 - The Demand for Capital



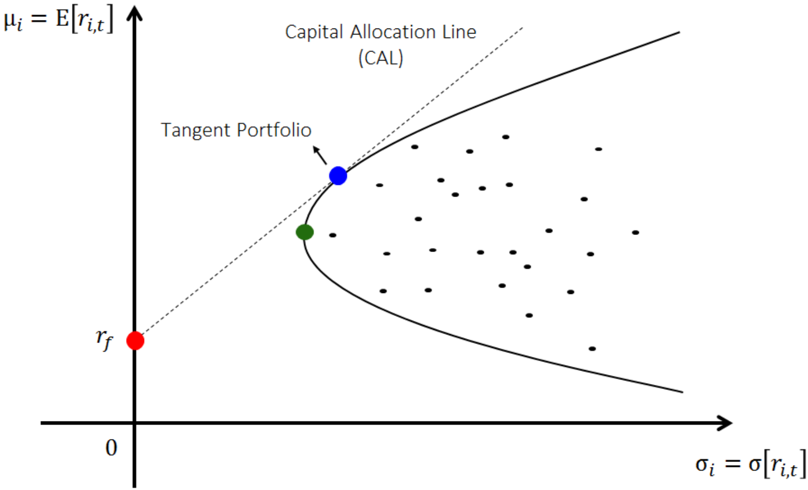
Module 1 - The Supply of Capital



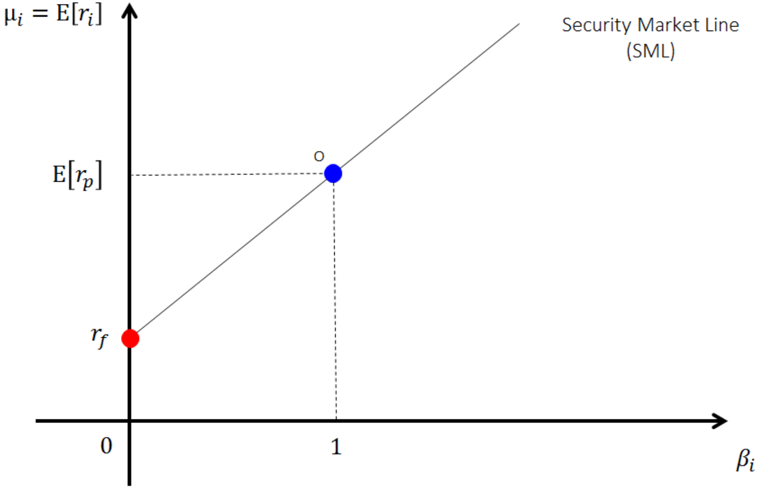
Module 1 - Investment Principle

$$PV_t = \sum_{h=1}^{\infty} \frac{\mathbb{E}_t \left[CF_{t+h} \right]}{\left(1 + dr_{t,h}\right)^h}$$

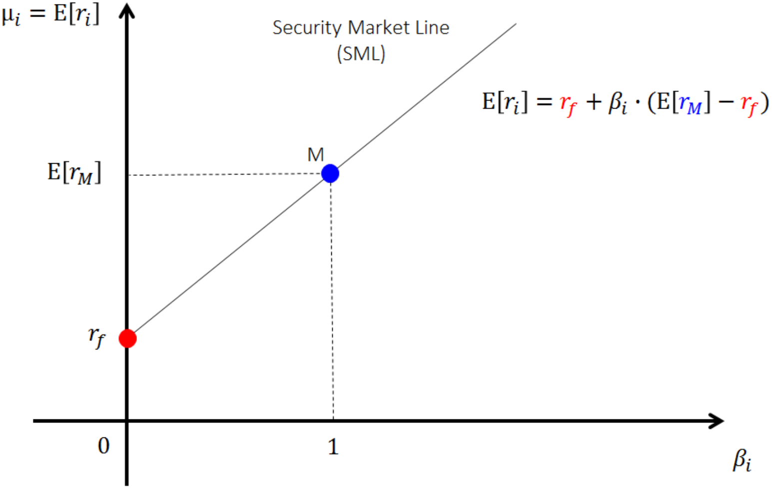
Module 2 - Portfolio Theory



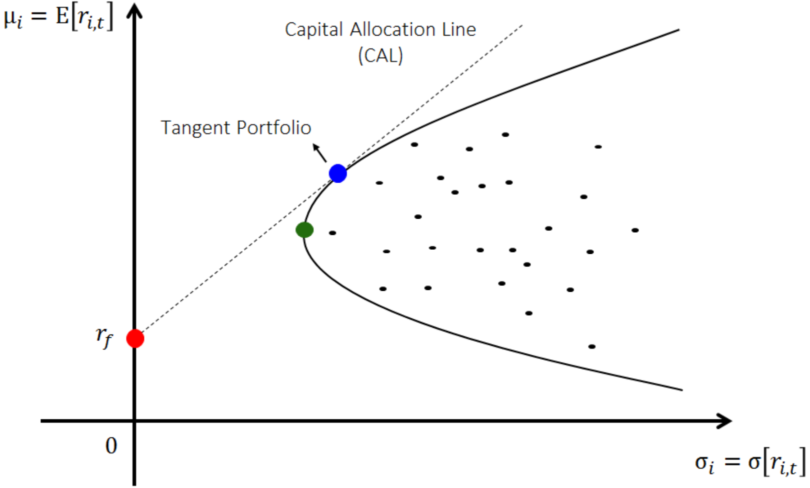
This Module: Factor Models



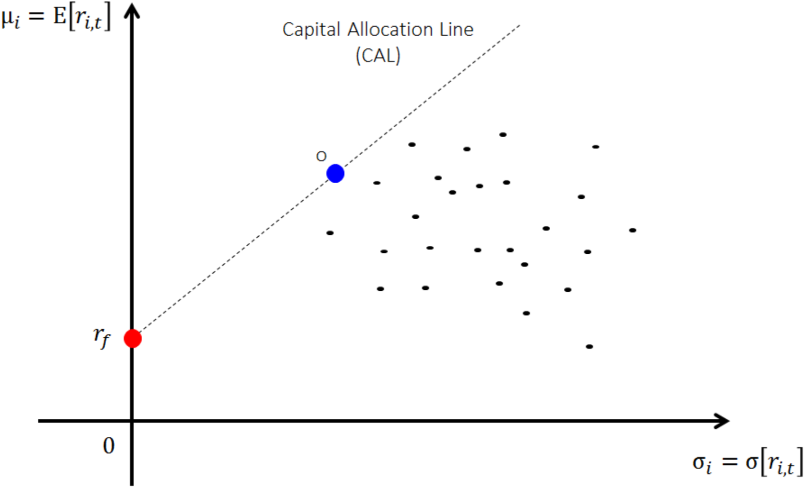
This Section: CAPM



Capital Allocation Line



Capital Allocation Line



Asset i Contribution to Portfolio Reward and Risk

$$
\begin{aligned}
\mathbb{E}[r_o] - r_f &= \mathbb{E}[w_1 \cdot r_1 + \ldots + w_N \cdot r_N] - r_f \\
&= \underbrace{w_1 \cdot (\mathbb{E}[r_1] - r_f)}_{\text{Contribution of Asset } 1} + \ldots + \underbrace{w_N \cdot (\mathbb{E}[r_N] - r_f)}_{\text{Contribution of Asset } N}
\end{aligned}
$$

$$
\begin{aligned}
\sigma^2[r_o] &= Cov[r_o, r_o] \\
&= Cov[w_1 \cdot r_1 + \ldots + w_N \cdot r_N, r_o] \\
&= \underbrace{w_1 \cdot Cov[r_1, r_o]}_{\text{Contribution of Asset } 1} + \ldots + \underbrace{w_N \cdot Cov[r_N, r_o]}_{\text{Contribution of Asset } N}
\end{aligned}
$$

Asset i Contribution to Portfolio Reward-to-Risk Ratio

- Since investors want to maximize $\left(\mathbb{E}[r_o] - r_f\right)/\sigma^2[r_o]$, the contribution of each asset to this expression must be the same:

$$\frac{\mathbb{E}\left[r_o\right] - r_f}{\sigma^2\left[r_o\right]} = \frac{w_i \cdot \left(\mathbb{E}\left[r_i\right] - r_f\right)}{w_i \cdot Cov\left[r_i, r_o\right]}$$

$$= \frac{\mathbb{E}\left[r_i\right] - r_f}{Cov\left[r_i, r_o\right]}$$

$$\Downarrow$$

$$\mathbb{E}\left[r_i\right] - r_f = \frac{Cov\left[r_i, r_o\right]}{\sigma^2\left[r_o\right]}\left(\mathbb{E}\left[r_o\right] - r_f\right)$$

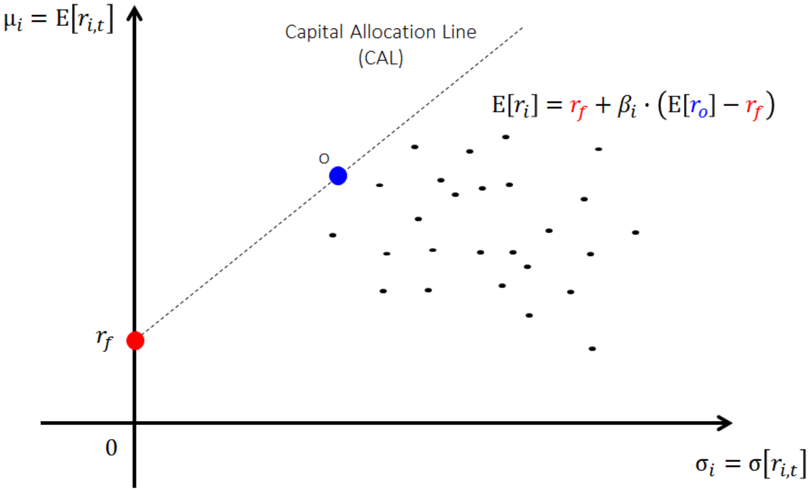
$$= \beta_i \cdot \left(\mathbb{E}\left[r_o\right] - r_f\right)$$

Explaining the Previous Slide

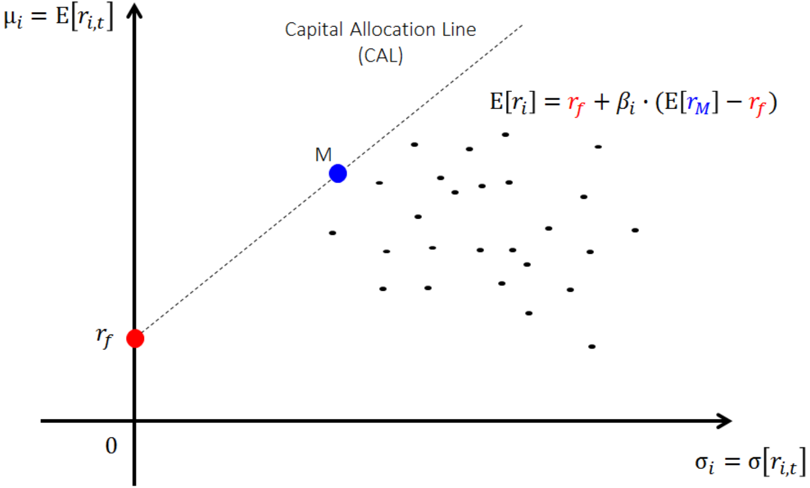
We learned in portfolio theory that the tangent portfolio is the maximum Sharpe Ratio portfolio. This is the same as saying that the tangent portfolio has the maximum possible value of $\frac{\mathbb{E}[r_o]-r_f}{\sigma^2[r_o]}$ (the Sharpe Ratio has volatility in the denominator instead). Let's call this expression the reward-to-risk ratio.

We saw two slides back that asset i contribution to the numerator of the reward-to-risk ratio is $w_i \cdot (\mathbb{E}[r_i] - r_f)$. Similarly, the contribution of asset i to the denominator of the reward-to-risk ratio is $w_i \cdot Cov[r_i, r_o]$. The only way to guarantee that the reward-to-risk ratio is at its maximum value is to assure that asset i contribution to the numerator relative to its contribution to the denominator is exactly identical to the reward-to-risk ratio. This is precisely what the first equation in the previous slide is saying. The rest is just algebra to find an expression for expected returns (we will dig into this expression further in the next slides).

CAPM: The Argument



CAPM: The Argument



Explaining the Previous Slide

We have an expression for expected returns: $\mathbb{E}[r_i] - r_f = \beta_i \cdot (\mathbb{E}[r_o] - r_f)$. However, this expression is only useful if we know what the tangent portfolio is.

The previous slide is providing you an argument for why the tangent portfolio is equal to the market portfolio: $r_o = r_M$.

The idea is pretty simple. If all investors use portfolio theory, have no restrictions in how they can invest and perceive markets identically (that is, use the same inputs to the portfolio problem), then they all end up with the same tangent portfolio. Well, if everybody holds the same portfolio, then this portfolio must contain all assets and they must be held in proportion to their market value. As such, we can substitute $r_o = r_M$ into $\mathbb{E}[r_i] - r_f = \beta_i \cdot (\mathbb{E}[r_o] - r_f)$, which gives the two key predictions of the CAPM (and they are intrinsically related):

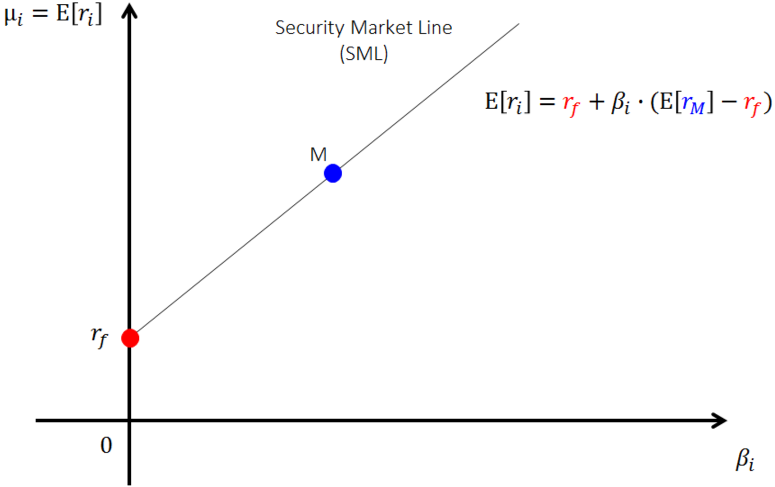
1. The market portfolio is efficient (in fact, it is the tangent portfolio)
2. $\mathbb{E}[r_i] - r_f = \beta_i \cdot (\mathbb{E}[r_M] - r_f)$ with $\beta_i = {}^{Cov(r_i, r_M)}/_{\sigma^2[r_M]}$

CAPM: Understanding the key Prediction

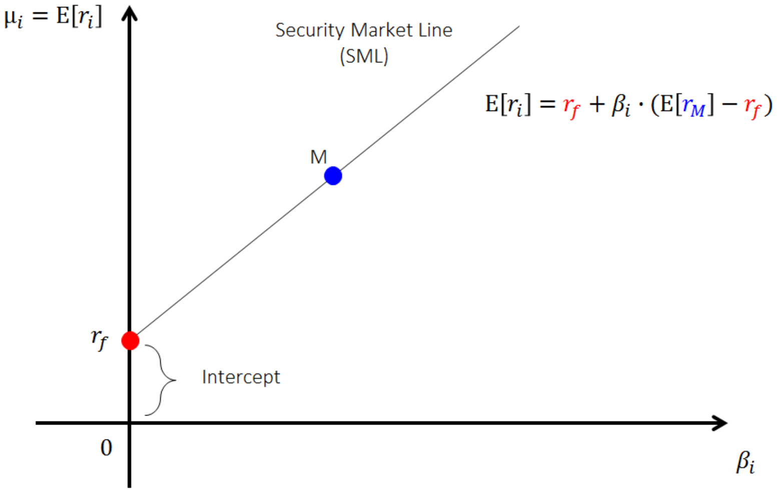
$$\mathbb{E}[r_i] = \underbrace{r_f}_{Risk-Free\ Reward} + \underbrace{\beta_i}_{Risk\ Exposure} \cdot \underbrace{\left(\mathbb{E}[r_M] - r_f\right)}_{Risk\ Premium}$$

- $\beta_i = {}^{Cov(r_i, r_M)}/_{\sigma^2[r_M]}$
- $\uparrow \beta_i \implies \uparrow \mathbb{E}[r_i]$:
- $\sigma^2[r]$ is not the right measure of risk. β is
- β_i controls security i contribution to $\sigma^2[r_M]$

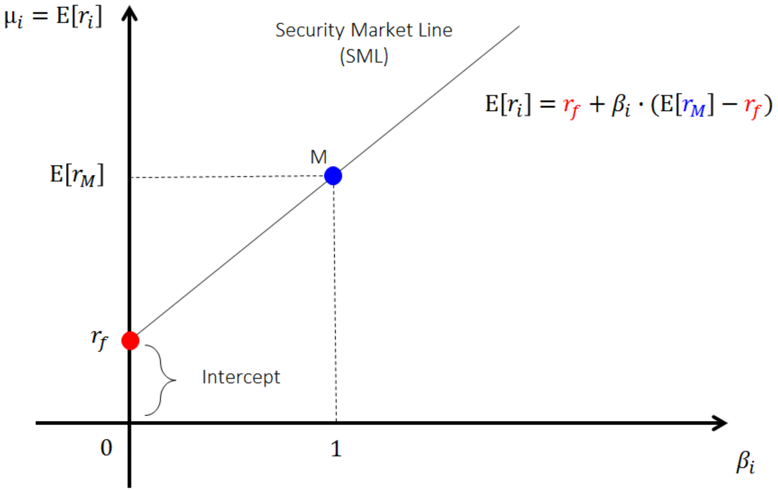
CAPM: Security Market Line



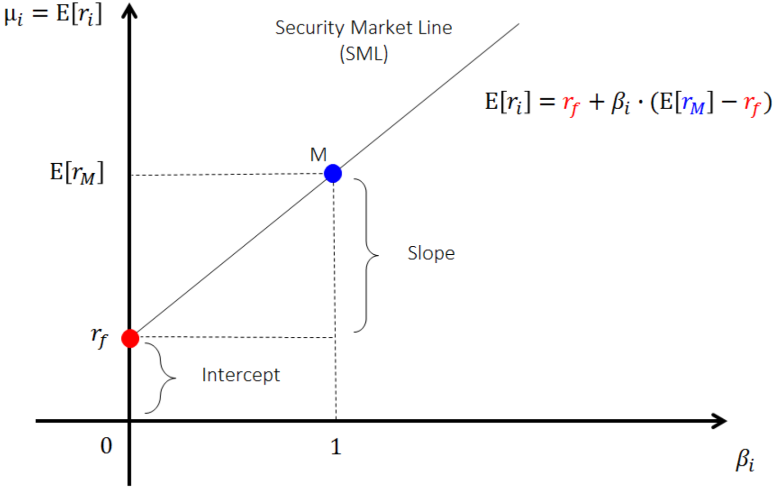
CAPM: Security Market Line



CAPM: Security Market Line



CAPM: Security Market Line



Explaining the Previous Slide

The Security Market Line (SML) is a simple geometric way to see the key prediction of the CAPM: $\mathbb{E}[r_i] = r_f + \beta_i \cdot (\mathbb{E}[r_M] - r_f)$. The SML treats the $\mathbb{E}[r_i]$ as our "y" variable and β as our "x" variable. Hence, it gives $\mathbb{E}[r_i]$ as a (linear) function of β. The linear function has slope given by $\mathbb{E}[r_M] - r_f$.

Reading the equation of the line, we have that each extra unit of risk, β, gives $\mathbb{E}[r_M] - r_f$ extra units of expected return to any asset. Hence, $\mathbb{E}[r_M] - r_f$ is the "market risk-premium" and β is the "amount of risk" or (market or systematic) "risk exposure" of asset i. Moreover, the risk-premium of asset i is given by its (market or systematic) risk exposure multiplied by the market risk-premium

CAPM: The Risk-Premium

$$\mathbb{E}\left[r_i\right] = \underbrace{r_f}_{Risk-Free\ Reward} + \underbrace{\beta_i}_{Risk\ Exposure} \cdot \underbrace{\left(\mathbb{E}\left[r_M\right] - r_f\right)}_{Risk\ Premium}$$

- When investors decide on their complete portfolios by maximizing their "Happiness" then: $\mathbb{E}\left[r_M\right] - r_f = A \cdot \sigma_M^2$
- This means that the Risk Premium:
  - Increases with market volatility:

    $$\uparrow \sigma_M \quad \Rightarrow \quad \uparrow \mathbb{E}\left[r_M\right] - r_f$$

  - Increases as investors get more risk averse:

    $$\uparrow A \quad \Rightarrow \quad \uparrow \mathbb{E}\left[r_M\right] - r_f$$

Suppose the CAPM holds: $\mathbb{E}[r_i] = r_f + \beta_i \cdot (\mathbb{E}[r_M] - r_f)$. Consider two stocks, A and B, with $\mathbb{E}[r_A] > \mathbb{E}[r_B]$ at time t. Between t and $t+1$ there is an increase in the volatility of the market portfolio, which induces an increase the market Risk-Premium (nothing else changes). What can you say about the expected return gap $\mathbb{E}[r_A] - \mathbb{E}[r_B]$?

a) It will increase from t to $t+1$

b) It will decrease from t to $t+1$.

c) It will remain the same from t to $t+1$ (β's did not change).

d) It will revert (become negative) from t to $t+1$ since stock A will be hit harder.

e) It will become zero at $t+1$.

CAPM: Relation to Index Model

$$\text{Index Model}: \quad r_{i,t} - r_f = \alpha_i + \beta_i \cdot (r_{M,t} - r_f) + e_{i,t}$$

$$\text{and}$$

$$\text{CAPM}: \quad \mathbb{E}[r_i] - r_f = \beta_i \cdot (\mathbb{E}[r_M] - r_f)$$

$$\Downarrow$$

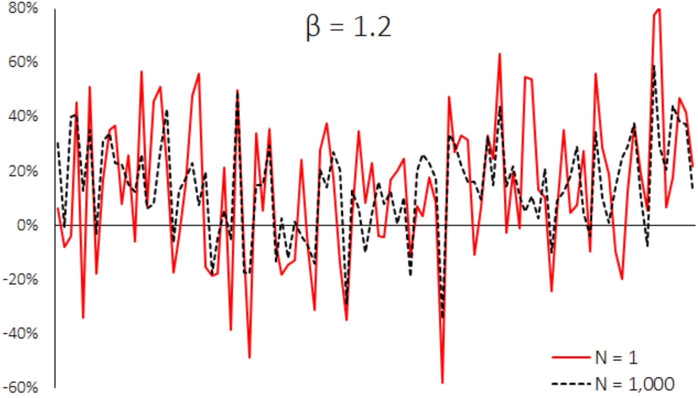
$$\alpha_i = 0$$

- The usual systematic x firm-specific risk decomposition holds:

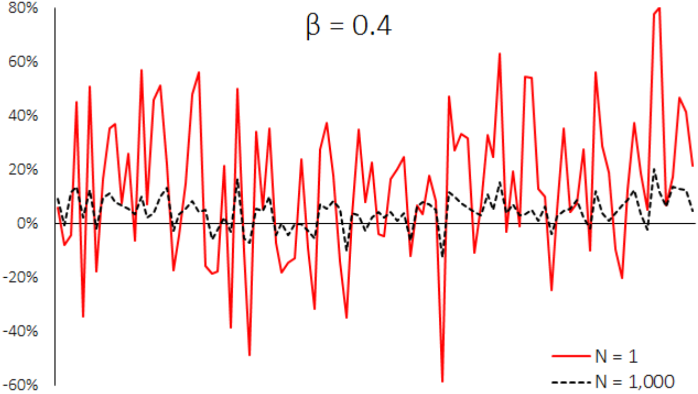
$$\sigma^2[r_i] = \beta_i^2 \cdot \sigma^2[r_M] + \sigma^2[e_i]$$

- We can estimate β using as the slope of a regression with $y = r_{i,t} - r_f$ and $x = r_{M,t} - r_f$

CAPM: β Effect on Portfolio Volatility



CAPM: β Effect on Portfolio Volatility



CAPM: Applications

- There are several CAPM applications, but two are particularly important since they are often used by market participants
- Portfolio Management: Risk Adjusted Returns

$$\mathbb{E}[r_i] - r_f = \alpha_i + \beta_i \cdot (\mathbb{E}[r_M] - r_f)$$

- You can use the simplest approach:

$$\widehat{\alpha}_i = \overline{r_i - r_f} - \widehat{\beta}_i \cdot \overline{r_M - r_f}$$

- Or you can use sophisticated security analysis:

$$\widehat{\alpha}_i = \widehat{\mathbb{E}}[r_i - r_f] - \widehat{\beta}_i \cdot \widehat{\mathbb{E}}[r_M - r_f]$$

CAPM: Applications

- There are several CAPM applications, but two are particularly important since they are often used by market participants

- Net Present Value Applications: Discount Rate

$$PV_t = \sum_{h=1}^{T} \frac{\mathbb{E}_t\left[CF_{t+h}\right]}{\left(1+dr_{t,h}\right)^h}$$

- The CAPM provides an estimate for the discount rate:

$$\begin{aligned}
\left(1+dr_{t,h}\right)^h &= \left(1+\widehat{\mathbb{E}}\left[r_i\right]\right)^h \\
&= \left(1+r_f+\widehat{\beta}_i \cdot \widehat{\mathbb{E}}\left[r_M - r_f\right]\right)^h \\
&\cong \left(1+r_f+\widehat{\beta}_i \cdot 6\%\right)^h
\end{aligned}$$

CAPM: The Paradox

- For the CAPM to hold, we need active investors in the market (using Portfolio Theory)
- But the main prediction of the CAPM is that the market portfolio is efficient
- Most people can easily buy an ETF that mimics the (stock) market portfolio
- There is no risk-adjusted return from being active. But there are costs (assumed away by the theory)
- Why would anyone be an active investor if the CAPM were true?

CAPM: The (false) Assumptions

- Complete Agreement (or "homogeneous expectations")!
- No private information!
- No taxes!
- Unlimited borrowing at the risk-free rate!
- No costs on transactions or information gathering/processing!
- Investors have same (single period) horizon!
- Investors are rational and use Portfolio Theory!
- For applications, we also need to have the market portfolio...

Use the CAPM in a Sensible way

- There are too many (false) assumptions for the CAPM to hold
- The question you should ask is not whether the CAPM holds or not. Instead you should ask: when is it reasonable to use it?
- As any model in Finance, when you blindly apply the CAPM you might face serious issues: mismeasure risk, mismeasure expected return, invest in unreasonable projects...
- Researchers have been working on improving the CAPM over more than three decades. Much progress has been made.
- Yet, most models keep the key insight from the CAPM: systematic risk matters!

Which of the following statements is false regarding the CAPM?

a) The CAPM links $\mathbb{E}[r_i]$ and systematic risk, measured by $\beta_i = {}^{Cov[r_i,r_M]}/_{\sigma^2[r_M]}$, directly. It says that asset A has higher expected return than asset B if and only if $\beta_A > \beta_B$

b) It is possible to have a world in which all investors use Portfolio Theory to decide on their portfolios and, yet, the CAPM predictions are false

c) Within the CAPM, the best risk measure for an asset or portfolio is its volatility: $\sigma^2[r]$

d) The CAPM prediction for $\mathbb{E}[r_i]$ can be used as a discount rate to be applied in Net Present Value (NPV) applications

e) The CAPM requires investors to use the same inputs when optimizing their portfolios

This Section: APT

"No free lunch" rule in Wall Street (+ index model) implies:

$$\mathbb{E}[r_p] = r_f + \beta_p \cdot (\mathbb{E}[r_M] - r_f)$$

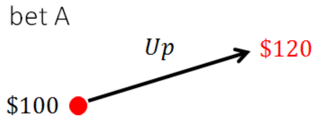
for any "well diversified" portfolio p

No Abitrage Principle

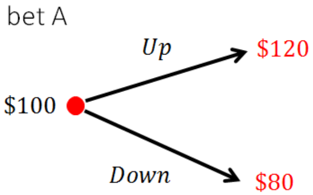
bet A

$100

No Abitrage Principle



No Abitrage Principle



No Abitrage Principle

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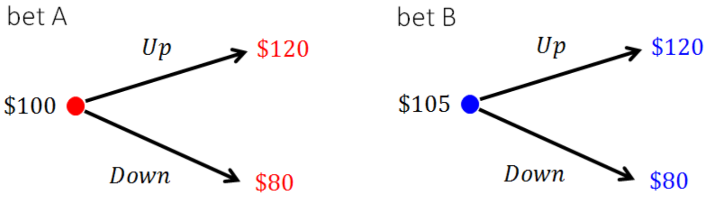

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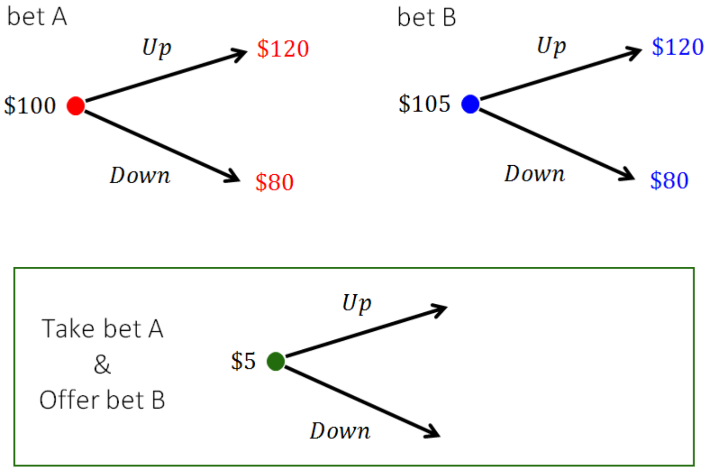

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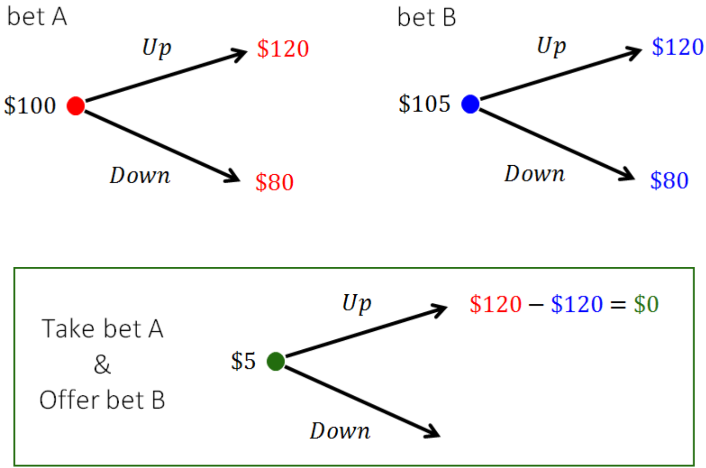

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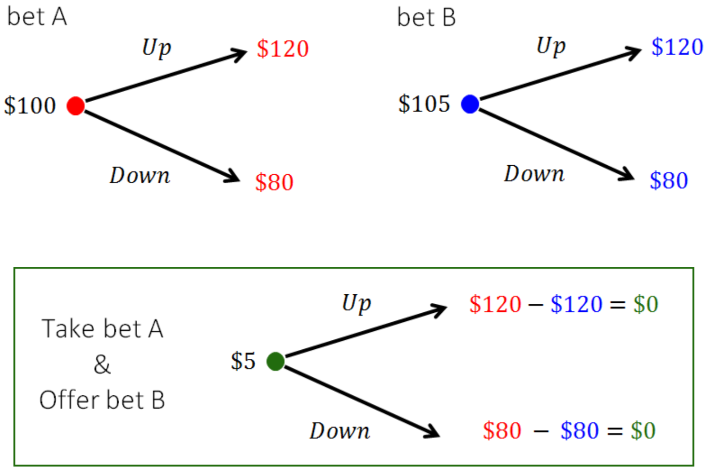

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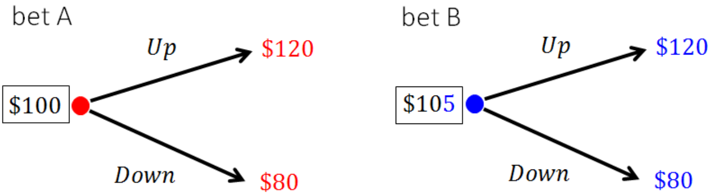
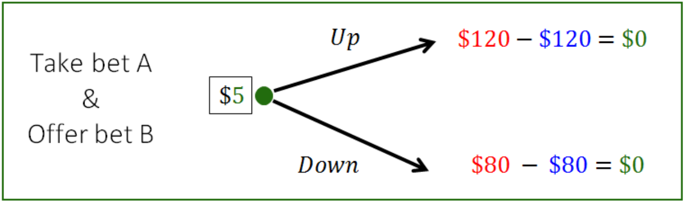




No Abitrage Principle

bet A

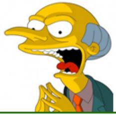
$\boxed{\$100}$ — *Up* → $120

$\boxed{\$100}$ — *Down* → $80

bet B

$\boxed{\$105}$ — *Up* → $120

$\boxed{\$105}$ — *Down* → $80



Take bet A

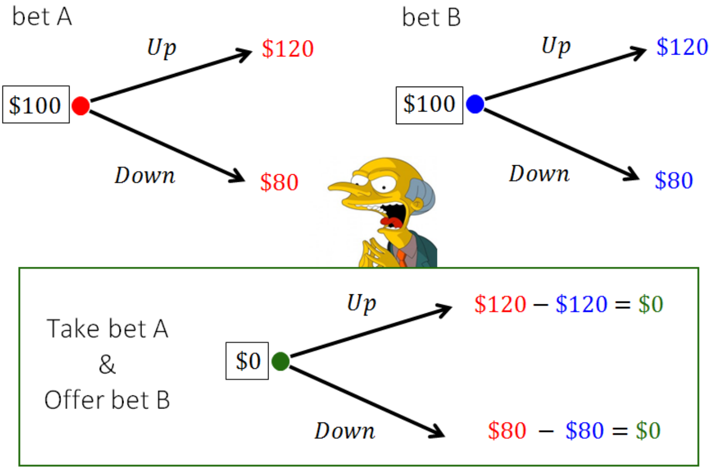
&

Offer bet B

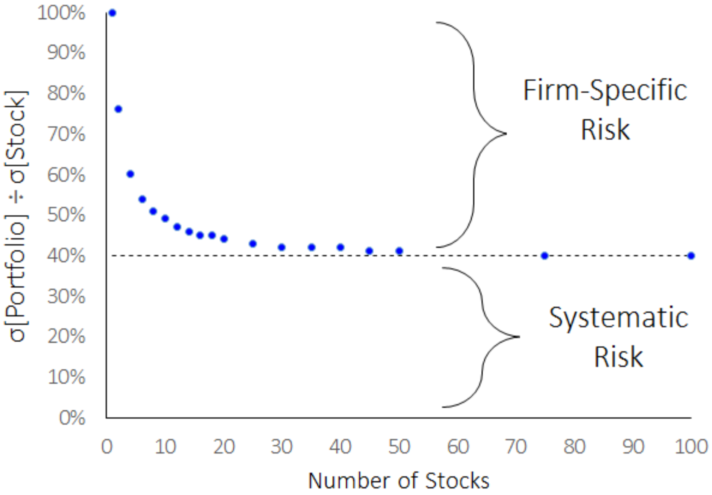
$\boxed{\$5}$ — *Up* → $\$120 - \$120 = \$0$

$\boxed{\$5}$ — *Down* → $\$80 - \$80 = \$0$

No Abitrage Principle



Systematic × Firm-Specific Risk



Source: "Statman (1987) - *How many stocks make a diversified portfolio*"

Systematic $\times$ Firm-Specific Risk

- Assume an index model holds:

$$r_{i,t} - r_f = \alpha_i + \beta_i \cdot (r_{M,t} - r_f) + e_{i,t}$$

$$\Downarrow$$

$$\sigma^2 [r_{i,t}] = \underbrace{\beta_i^2 \cdot \sigma^2 [r_{M,t}]}_{systematic\ risk} + \underbrace{\sigma^2 [e_{i,t}]}_{firm-specific\ risk}$$

Firm-Specific Risk Vanishes in Well Diversified Portfolios

- When we form a (equal-weighted) portfolio $r_p = \frac{1}{N}\sum_{i=1}^{N} r_i$:

$$r_p - r_f = \underbrace{\left(\frac{1}{N}\sum_i \alpha_i\right)}_{\alpha_p} + \underbrace{\left(\frac{1}{N}\sum_i \beta_i\right)}_{\beta_p} \cdot (r_M - r_f) + \left(\frac{1}{N}\sum_i e_i\right)$$

$$= \alpha_p + \beta_p \cdot (r_M - r_f) + \left(\frac{1}{N}\sum_i e_i\right)$$

$$\Downarrow$$

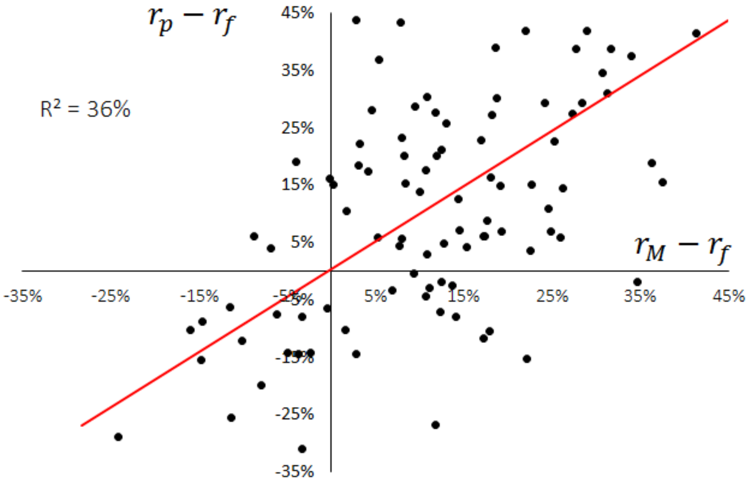
$$r_p - r_f \cong \alpha_p + \beta_p \cdot (r_M - r_f)$$

Explaining the Previous and Next Slides

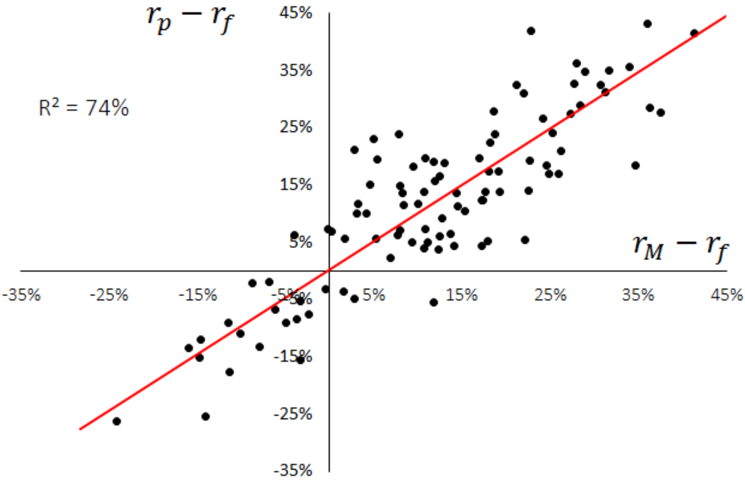
If all assets follow an index model, then forming a large portfolio means that we do not have any firm specific risk. The reason is that movements in the firm-specific component, e_i, cancel out and, thus, $\frac{1}{N}\sum_i e_i$ approaches zero as we increase the number of assets in the portfolio. Even though I used an equal-weighted portfolio, the argument holds true for any weighting scheme as long as no asset has a large weight that never decreases no matter how many assets we add to the portfolio.

The next slide is simply providing a picture that was generated with simulations in order to demonstrate the decrease in firm-specific risk as we increase the number of assets in the portfolio. The figure is just a regression and the returns on the market become better in predicting the returns of the portfolio as we increase the number of assets in the portfolio. When there are 1,000 assets in the portfolio, all movements in the portfolio can be predicted (almost) perfectly by the market return and we have: $r_p - r_f \cong \alpha_p + \beta_p \cdot (r_M - r_f)$

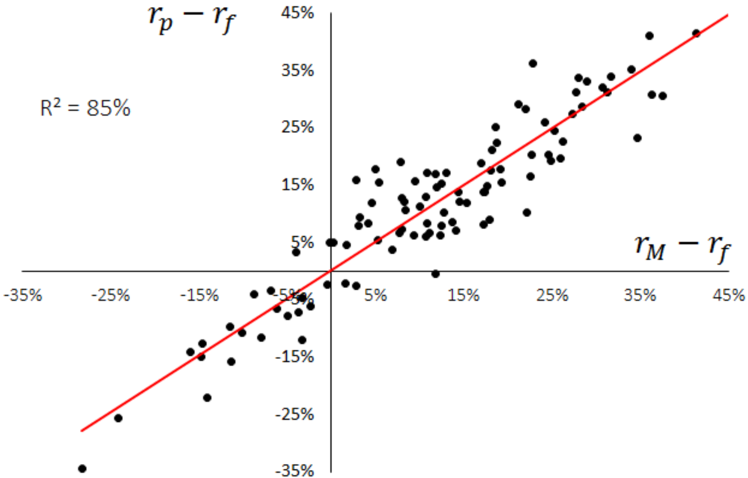
$N = 1$



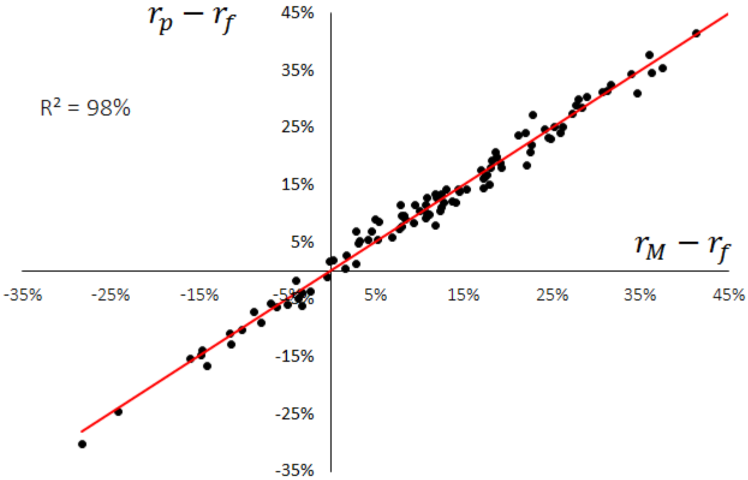
$N = 5$



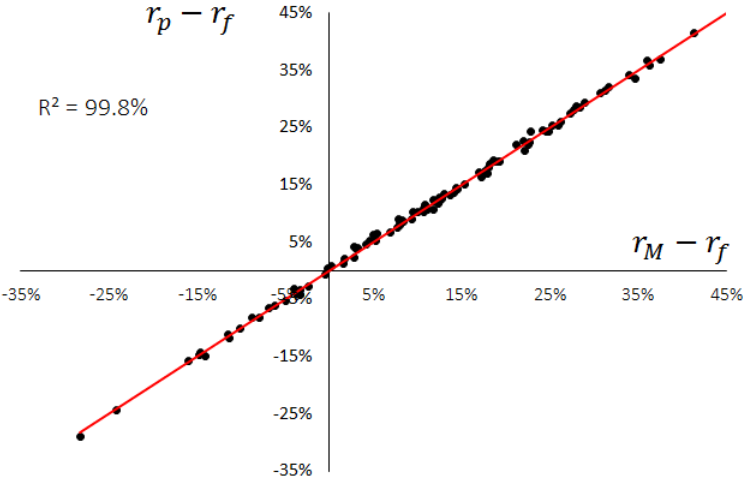
$N = 10$



$N = 100$



$N = 1,000$



Creating an Arbitrage Strategy

- For a well diversified portfolio, p, we have:

$$r_p - r_f = \alpha_p + \beta_p \cdot (r_M - r_f)$$
$$r_M - r_f = 0 + 1 \cdot (r_M - r_f)$$

- Let's create a portfolio, $r_z = w_p \cdot r_p + (1 - w_p) \cdot r_M$, with zero systematic risk:

$$\beta_z = w_p \cdot \beta_p + (1 - w_p) \cdot 1 = 0$$
$$\Downarrow$$
$$w_p = \frac{1}{1 - \beta_p}$$

Creating an Arbitrage Strategy

- Our portfolio, r_z, has $\beta_z = 0$ and $\alpha_z = w_p \cdot \alpha_p$. Hence:

$$\begin{aligned} r_z - r_f &= \alpha_z + \beta_z \cdot (r_M - r_f) \\ &= \alpha_z \\ &\Downarrow \\ r_z &= r_f + \alpha_z \end{aligned}$$

- We just created a risk-free asset paying an interest rate higher than the risk-free rate (lower if $\alpha_z < 0$)
- This cannot be sustainable (smart investors will arbitrage that difference away by taking contrary positions on the r_f and r_z)
- As a consequence, α_z is driven to zero. Of course, α_z depends only on α_p, which means that $\alpha_p = 0$

APT x CAPM

- CAPM (Equilibrium Principle):

$$\textit{Portfolio Theory}$$
$$\Downarrow$$
$$\mathbb{E}[r_i] = r_f + \beta_i \cdot (\mathbb{E}[r_M] - r_f)$$

- APT (Non-Arbitrage Principle):

$$r_{i,t} - r_f = \alpha_i + \beta_i \cdot (r_{M,t} - r_f) + e_{i,t}$$
$$\Downarrow$$
$$\mathbb{E}[r_p] = r_f + \beta_p \cdot (\mathbb{E}[r_M] - r_f)$$

Explaining the Previous Slide

There are two principles in financial economics that give rise to factor models.

The first is the "Equilibrium Principle". The idea there is that many investors act in a "competitive way" and the cumulative effect of their behavior has implications for what we observe in financial markets. The CAPM is based on such principle and the "competitive way" there is to use portfolio theory in order to choose the best possible risk-return combination.

The second is the "Non-Arbitrage Principle". The idea is that whenever there is an investment opportunity that allows an investor to make money without requiring any initial capital or risk bearing (i.e., a free lunch), then any investor who notices that will take a position that is so large that drives this opportunity away quickly. As a consequence, no such opportunity can be found in financial markets.

The key distinction is that the "Equilibrium Principle" requires the action of many investors and prices adjust over time while the "Non-Arbitrage Principle" requires only one investor and prices adjust quickly. There is a trade off, however. APT provides predictions for well-diversified portfolios (not for single assets) while the CAPM applies to both single assets and portfolios.

In principle, which of the following assumptions is necessary for the APT to work?

a) Many investors need to try to take advantage of arbitrage opportunities offered by deviations from the APT implications.

b) Investors must prefer lower risk, $\sigma^2[r]$, and higher reward, $\mathbb{E}[r]$.

c) Investors need to be homogeneous in the sense that they estimate $\sigma^2[r]$ and $\mathbb{E}[r]$ the same way.

d) At least one investor must have access to lending/borrowing at the risk-free and use this capacity to take advantage of arbitrage opportunities offered by deviations from the APT implications.

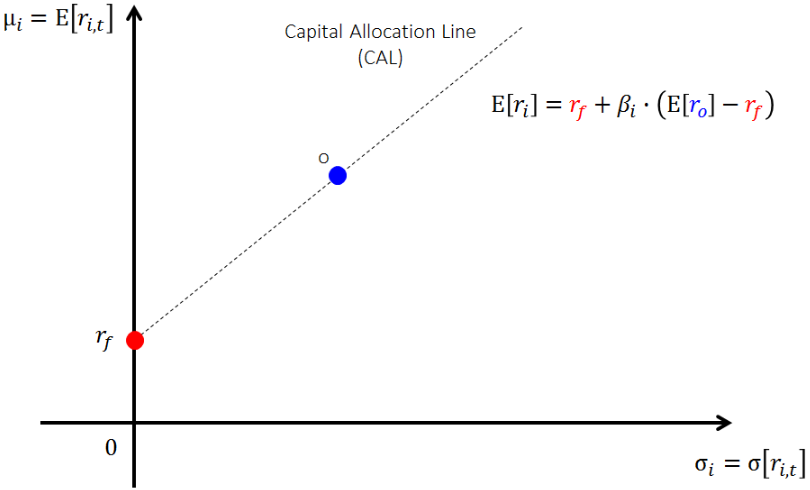
e) Investors must be rational and use Portfolio Theory.

This Section: Multiple Factors

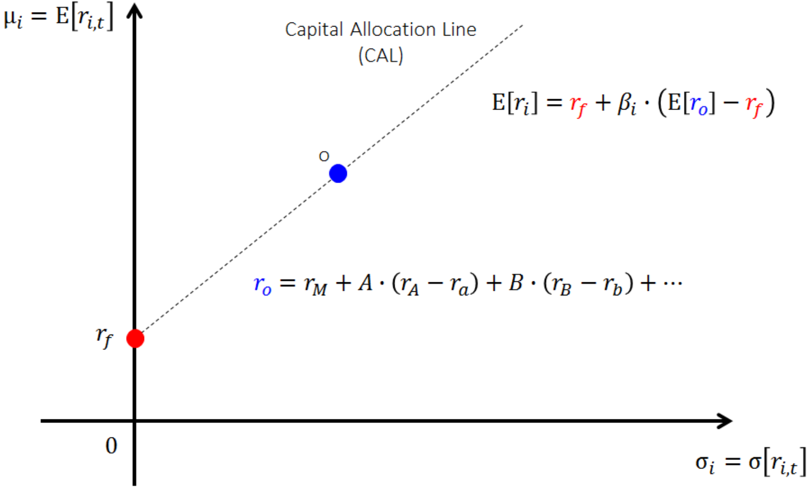
We can generalize the logic in the previous sections:

$$\begin{aligned}\mathbb{E}[r_i] = r_f + \beta_i \cdot (\mathbb{E}[r_M] - r_f) \\ + \beta_{i,A} \cdot \mathbb{E}[r_A - r_a] \\ + \beta_{i,B} \cdot \mathbb{E}[r_B - r_b] \\ + \ldots\end{aligned}$$

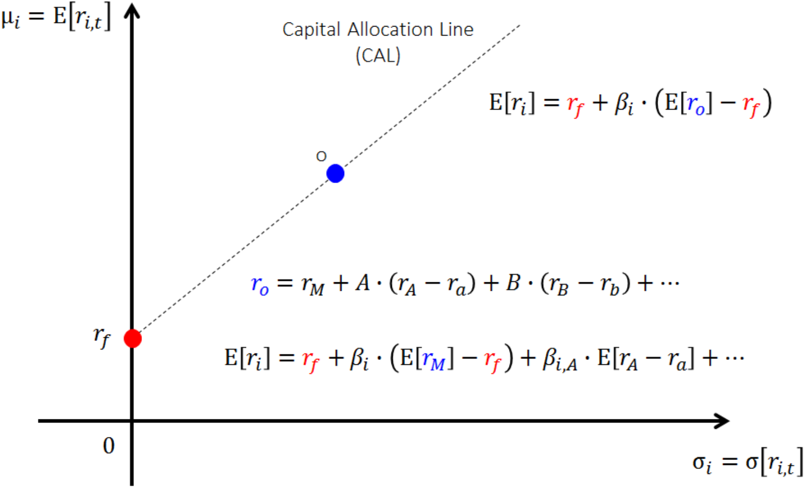
CAPM: Multifactor Equilibrium Models



CAPM: Multifactor Equilibrium Models



CAPM: Multifactor Equilibrium Models



Explaining the Previous Slide

We saw in the first section of this Module that $\mathbb{E}[r_i] - r_f = \beta_i \cdot (\mathbb{E}[r_o] - r_f)$ holds true even if the restrictive assumptions of the CAPM are not valid. The benefit of the CAPM is that it tells us what the tangent portfolio is: $r_o = r_M$.

Other equilibrium models give rise to alternative tangent portfolios. Multifactor models typically tell us that the tangent portfolio is composed by a passive position on the market portfolio plus several active bets in different Long-Short positions, $r_A - r_a$, $r_B - r_b$, ...

For instance, we will see in the next section that one such active position is a value strategy. That is, Long a portfolio of “value stocks" and short a portfolio of “growth stocks”. Many others have been studied.

The key is that the importance of these active positions in composing the tangent portfolio induces these excess returns to be new “factors” just like the market portfolio in the CAPM

$$\mathbb{E}[r_p] = r_f + \beta_p \cdot (\mathbb{E}[r_M] - r_f) + \beta_{p,A} \cdot \mathbb{E}[r_A - r_a] + \ldots$$

APT: Multifactor Arbitrage Pricing Models

$$r_{i,t} - r_f = \alpha_i + \beta_i \cdot (r_{M,t} - r_f) + \beta_{i,A} \cdot (r_A - r_a) + ... + e_{i,t}$$

$$+$$

No free lunch in Wall street

$$\Downarrow$$

$$\mathbb{E}[r_p] = r_f + \beta_p \cdot (\mathbb{E}[r_M] - r_f) + \beta_{p,A} \cdot \mathbb{E}[r_A - r_a] + ...$$

That is, for any "well diversified" portfolio: $\alpha_p = 0$

This Section: CAPM Tests and the 3 Factor Model

CAPM empirical tests indicate it is an inadequate model. However, its logic is relevant and the most commonly applied factor model was created from a careful analysis of the CAPM failure.

CAPM: SML Prediction

- From CAPM, we have:

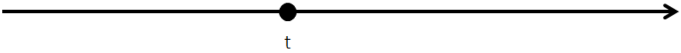
$$\mathbb{E}[r_i] = r_f + \beta_i \cdot (\mathbb{E}[r_M] - r_f)$$

$$\bar{r}_i = \lambda_0 + \widehat{\beta}_i \cdot \lambda_1 + \epsilon_i$$

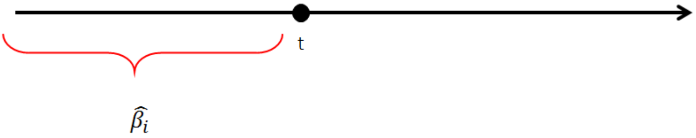
$$\Downarrow$$

$$\lambda_0 = r_f \quad \textit{and} \quad \lambda_1 = \mathbb{E}[r_M] - r_f$$

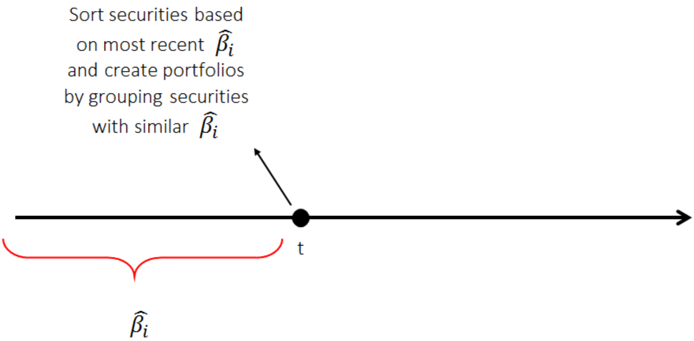
CAPM: $\hat{\beta}$ Sorted Portfolios



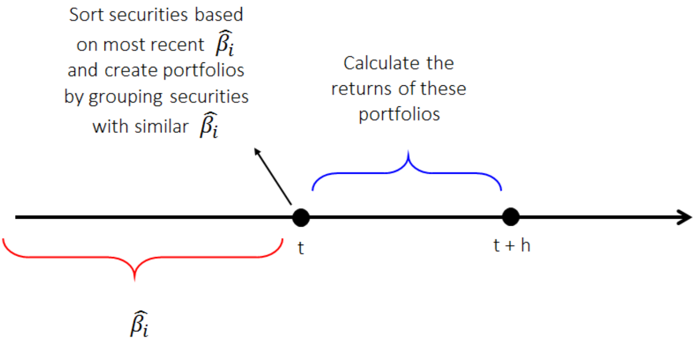
CAPM: $\hat{\beta}$ Sorted Portfolios



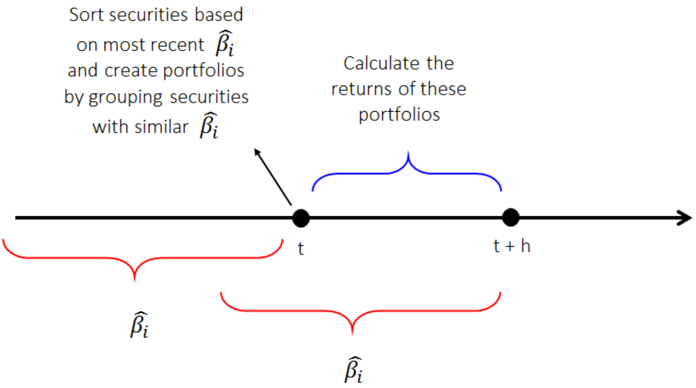
CAPM: $\hat{\beta}$ Sorted Portfolios



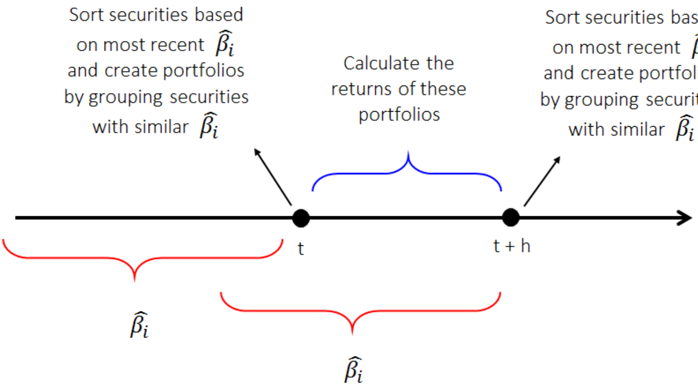
CAPM: $\hat{\beta}$ Sorted Portfolios



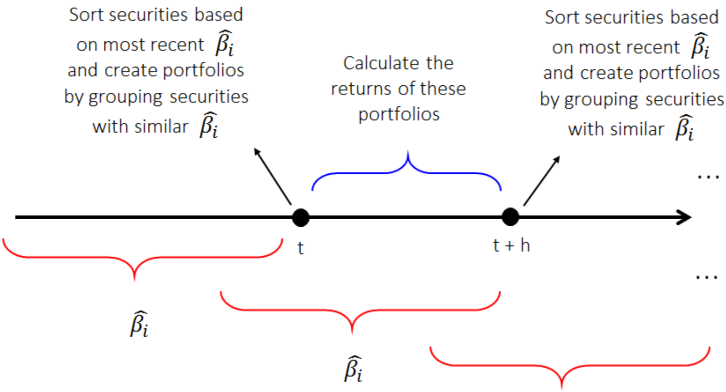
CAPM: $\hat{\beta}$ Sorted Portfolios



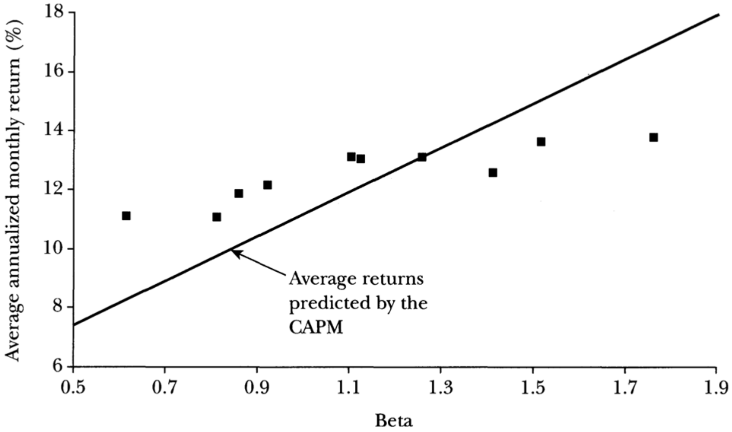
CAPM: $\hat{\beta}$ Sorted Portfolios



CAPM: $\hat{\beta}$ Sorted Portfolios

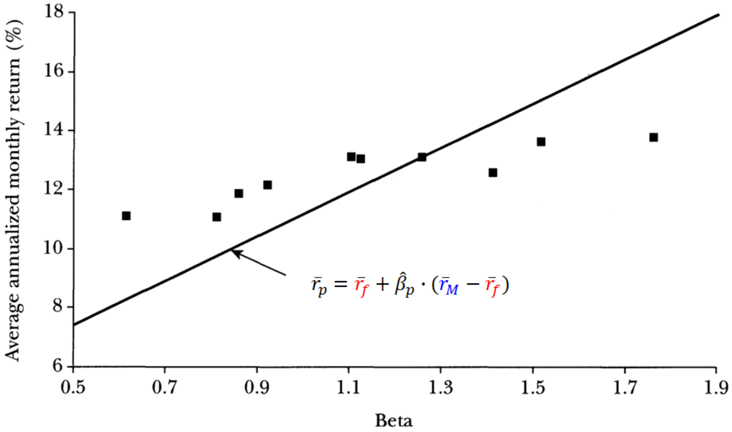


CAPM: SML from 1928 to 2003 (US Equity)



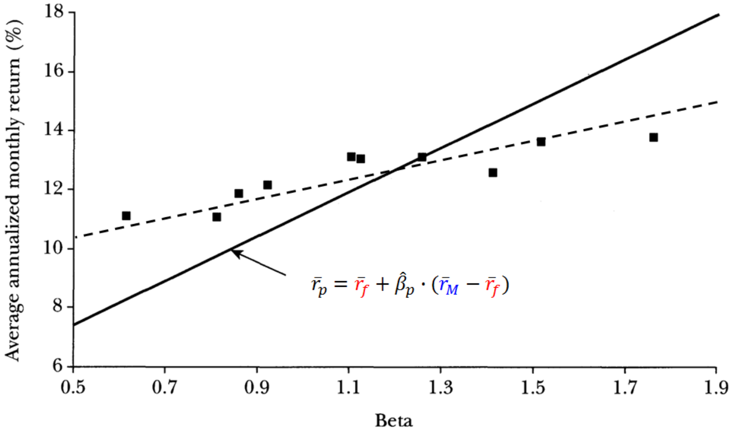
Source: Fama and French (2004) - *The Capital Asset Pricing Model: Theory and Evidence*

CAPM: SML from 1928 to 2003 (US Equity)



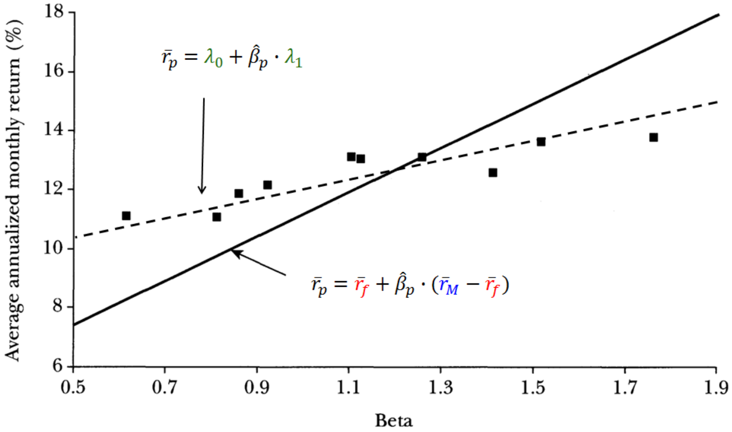
Source: Fama and French (2004) - *The Capital Asset Pricing Model: Theory and Evidence*

CAPM: SML from 1928 to 2003 (US Equity)



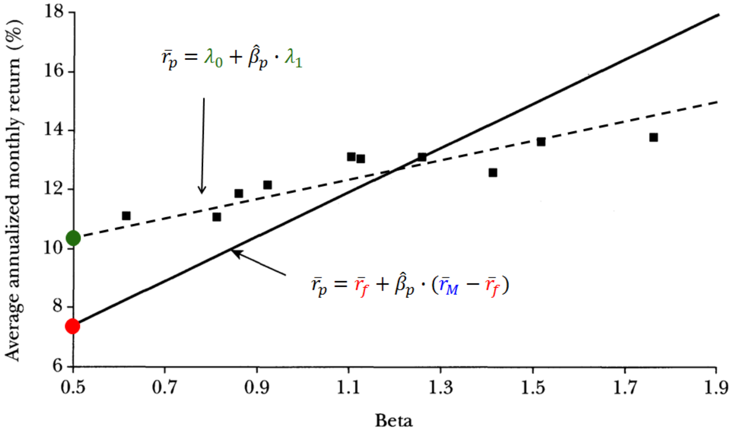
Source: Fama and French (2004) - *The Capital Asset Pricing Model: Theory and Evidence*

CAPM: SML from 1928 to 2003 (US Equity)



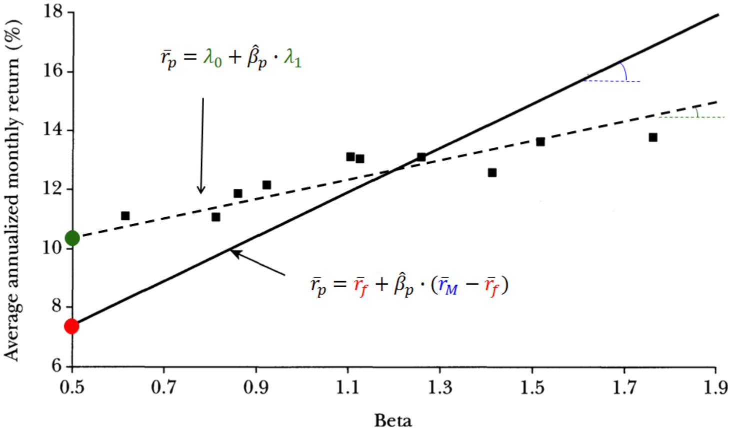
Source: Fama and French (2004) - *The Capital Asset Pricing Model: Theory and Evidence*

CAPM: SML from 1928 to 2003 (US Equity)



Source: Fama and French (2004) - *The Capital Asset Pricing Model: Theory and Evidence*

CAPM: SML from 1928 to 2003 (US Equity)



Source: Fama and French (2004) - *The Capital Asset Pricing Model: Theory and Evidence*

Explaining the Previous Slide

This graph plots the average return and β of 10 portfolios formed in the way detailed two slides back. The graph points out one success and one failure of the CAPM:

- Success: it is clear to see that average returns do increase with β's. Portfolios with higher β do in fact deliver higher rewards
- Failure: the key failure of the CAPM in this graph is that even though average returns increase in β, they do so with a slope that is lower than what the CAPM predicts. Recall that (based on the SML) the slope of the line linking expected returns to β is $\mathbb{E}[r_M] - r_f$ (the market risk premium). From the figure we see that the actual slope is lower than that. It is also true that the intercept is above the risk-free rate. Both facts are evidence against the CAPM (a least in its original form)

CAPM: α Prediction

- From CAPM, we have:

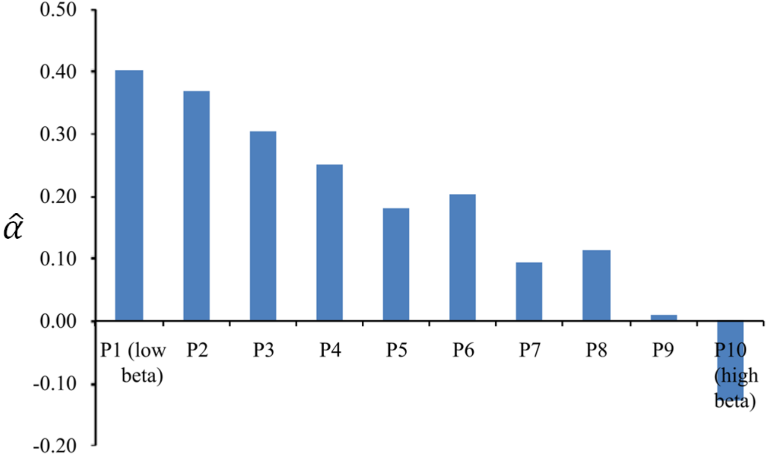
$$\mathbb{E}[r_i] - r_f = \beta_i \cdot (\mathbb{E}[r_M] - r_f)$$

$$r_{i,t} - r_f = \alpha_i + \beta_i \cdot (r_{M,t} - r_f) + e_{i,t}$$

$$\Downarrow$$

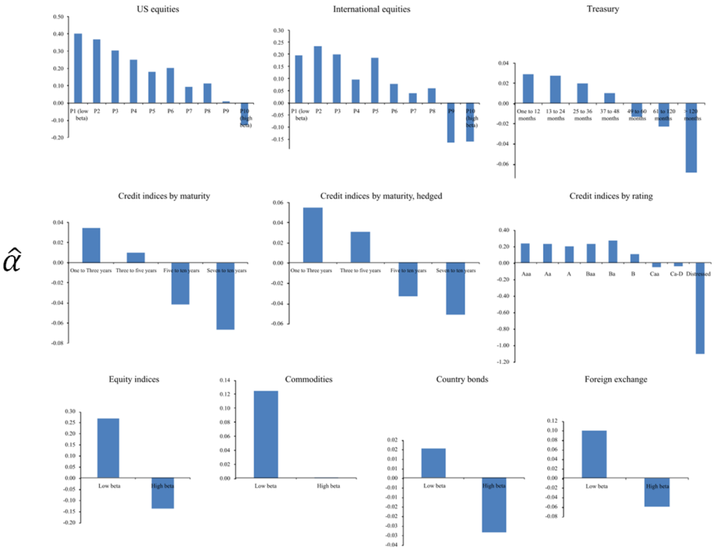
$$\alpha_i = 0$$

CAPM: $\hat{\alpha}$ from 1926 to 2012 (US Equity)



Source: Franzzini and Petersen (2014) - *Betting Against Beta*

CAPM: $\hat{\alpha}$ Across Asset Classes



Source: Franzzini and Petersen (2014) - *Betting Against Beta*

Explaining the Previous Slide

These graphs demonstrate that portfolios of stocks with low risk (β) tend to have much higher risk-adjusted return (α) than portfolios of stocks with high β. Each bar in the bar graph represents the α of the respective portfolio.

What is quite striking is that this is valid not only in the US equity market, but also in international markets and in markets for other financial assets.

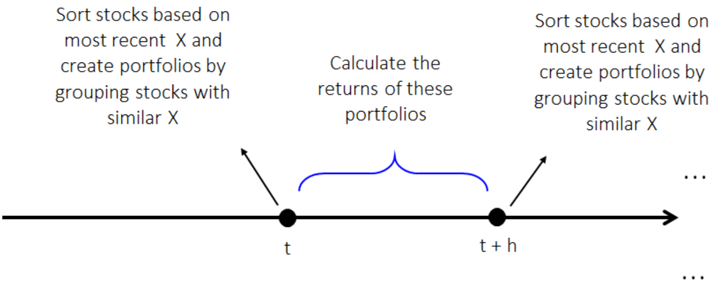
This represents a failure of the CAPM since $\alpha = 0$ is a prediction of the CAPM while we can find strong patterns in α when looking at the data

CAPM: "β is the only Risk Measure" Prediction

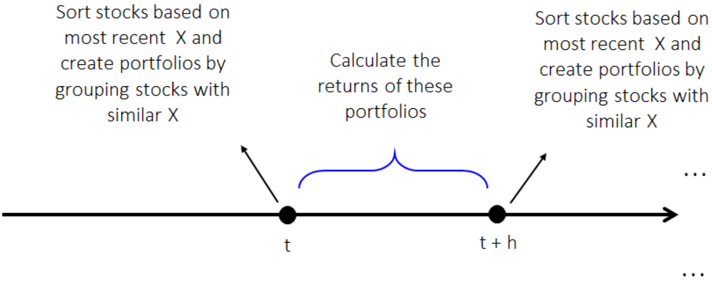
- From CAPM, we have:

$$\mathbb{E}[r_i] = r_f + \beta_i \cdot (\mathbb{E}[r_M] - r_f)$$

CAPM: Characteristic Sorted Portfolios



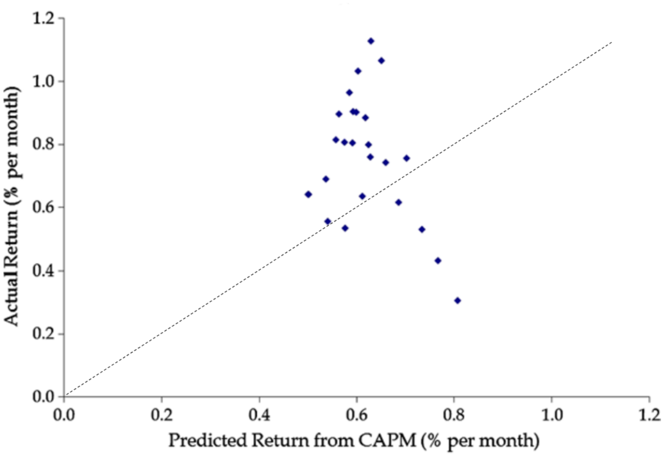
CAPM: Characteristic Sorted Portfolios



X = Size of Firm

X = Book Equity / Market

CAPM: Size x Book-to-Market Portfolios (1946 to 2010)



Source: Goyal (2012) - *Empirical Cross-Sectional Asset Pricing: a Survey*

Explaining the Previous Slide

In this graph, there are 25 portfolios formed by sorting stocks on Book-to-Market (a measure of "Value") and Market Equity (a measure of Size).

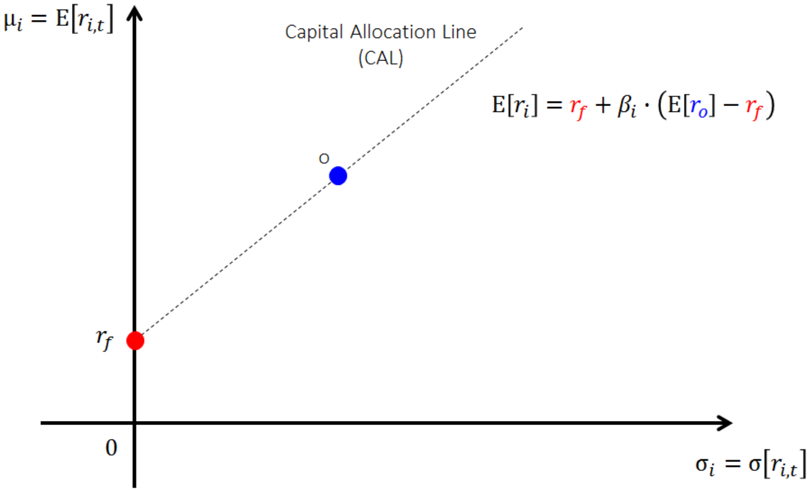
The graph plots the average return on those portfolios (y axis) against the average return that would be predicted by the CAPM (x axis):

$$\mathbb{E}\left[r_p\right] = r_f + \beta_p \cdot \left(\mathbb{E}\left[r_M\right] - r_f\right)$$

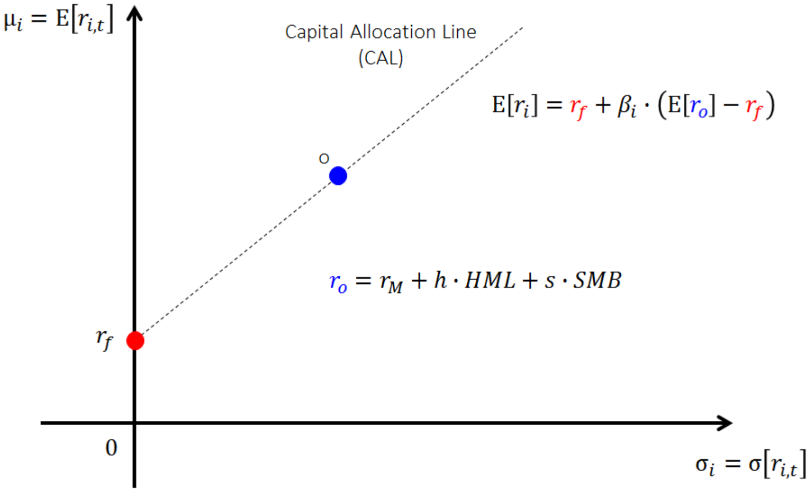
If the model were valid, then points would be (close to) the 45 degrees line plotted. However, they are very far from that, which indicates that the model fails strongly to predict the returns of small and value stocks

The last graph of this section (a few slides from now) demonstrates that an alternative factor model (called the 3-Factor model or the Fama-French model) does well in predicting the average returns of portfolios sorted on market equity and book-to-market

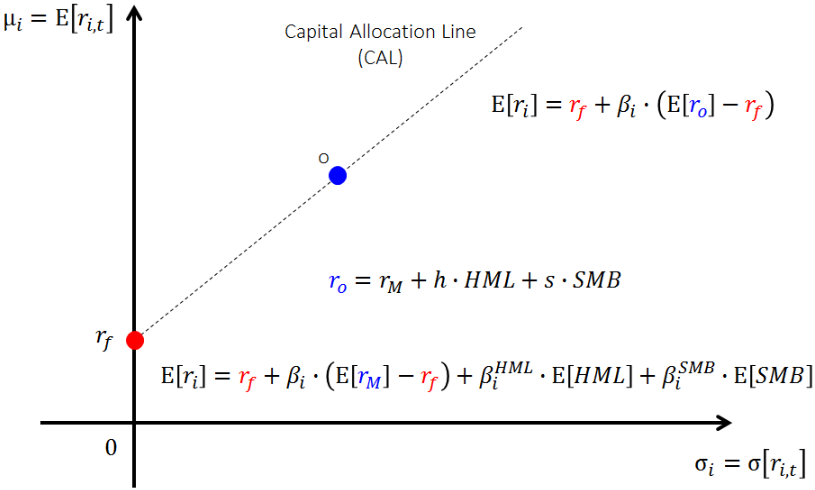
3 Factor Model: Equilibrium Justification



3 Factor Model: Equilibrium Justification



3 Factor Model: Equilibrium Justification



3 Factor Model: APT Justification

- Value stocks comove with other value stocks and growth stocks with other growth stocks. The same is true for small vs large companies. Therefore:

$$r_{i,t} - r_f = \alpha_i + \beta_i \cdot (r_{M,t} - r_f) + \beta_i^{HML} \cdot HML_t + \beta_i^{SMB} \cdot SMB_t + e_{i,t}$$

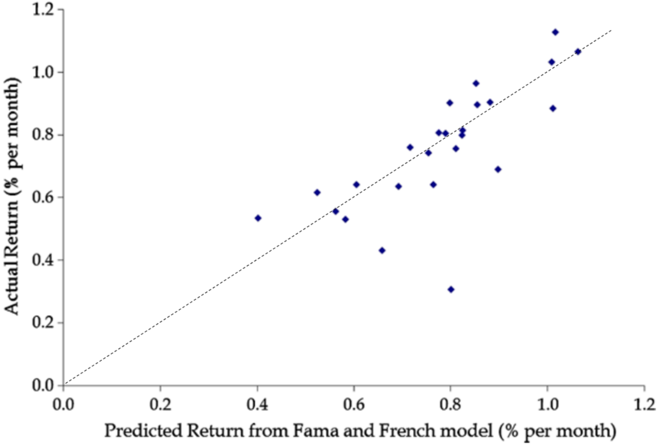
$$+$$

No free lunch in Wall street

$$\Downarrow$$

$$\mathbb{E}[r_p] = r_f + \beta_p \cdot (\mathbb{E}[r_M] - r_f) + \beta_p^{HML} \cdot \mathbb{E}[HML] + \beta_p^{SMB} \cdot \mathbb{E}[SMB]$$

3 Factor Model: Size x Book-to-Market Porfolios (1946 to 2010)



Source: Goyal (2012) - *Empirical Cross-Sectional Asset Pricing: a Survey*

From an empirical perspective, there are three key failures of the CAPM. Can you explain what are the failures and what is the evidence behind each one of them?

References