Module 5: Debt Securities

(BUSFIN 4221 - Investments)

Andrei S. Gonçalves[1]

[1]Finance Department
The Ohio State University

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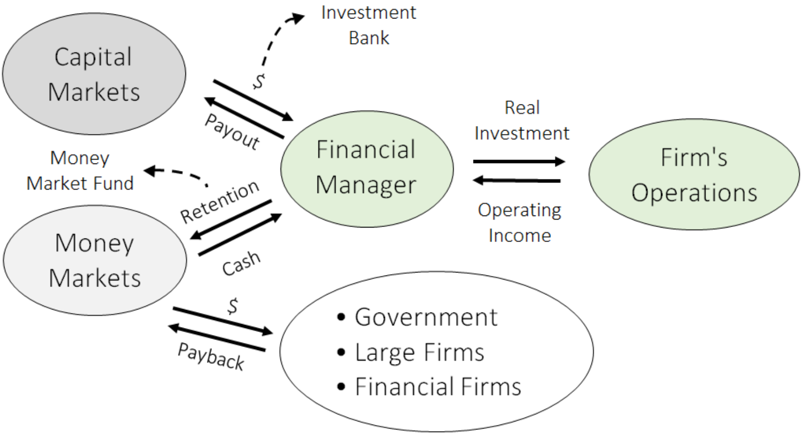
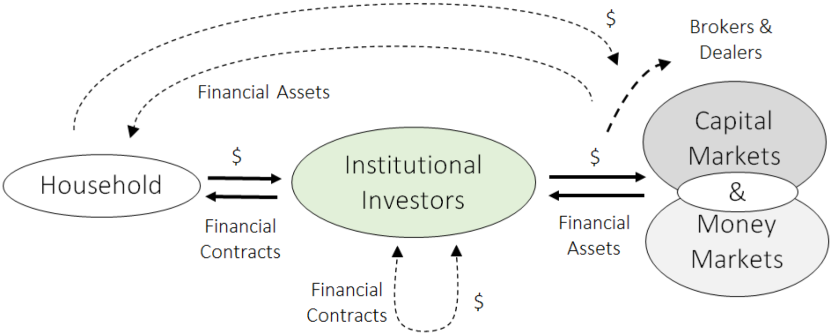
Module 1 - The Demand for Capital



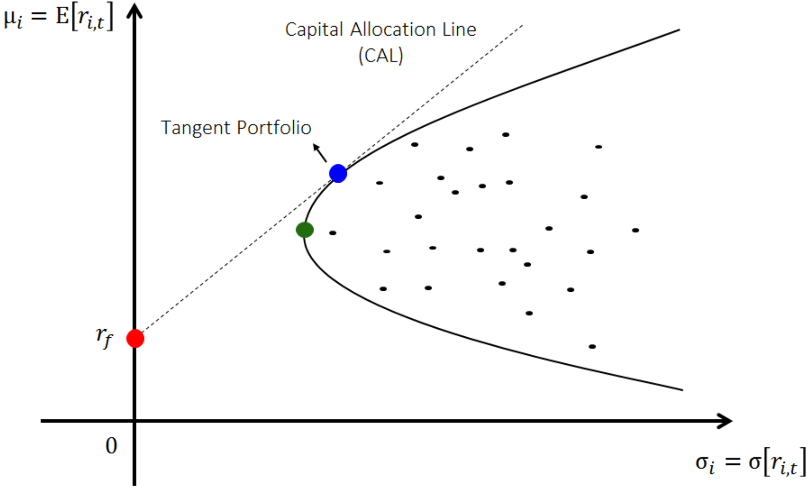
Module 1 - The Supply of Capital



Module 1 - Investment Principle

$$PV_t = \sum_{h=1}^{\infty} \frac{\mathbb{E}_t\left[CF_{t+h}\right]}{\left(1 + dr_{t,h}\right)^h}$$

Module 2 - Portfolio Theory



Module 3 - Factor Models

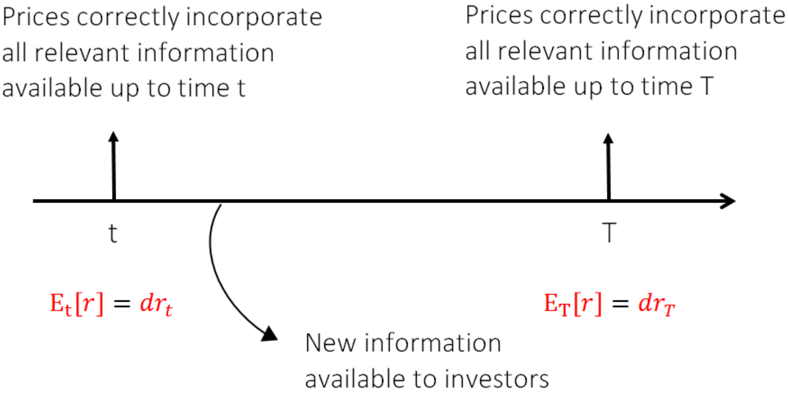
$$\mathbb{E}[r_i] = r_f + \beta_i \cdot (\mathbb{E}[r_M] - r_f)$$

$$+ \beta_{i,A} \cdot \mathbb{E}[r_A - r_a]$$

$$+ \beta_{i,B} \cdot \mathbb{E}[r_B - r_b]$$

$$+ \ldots$$

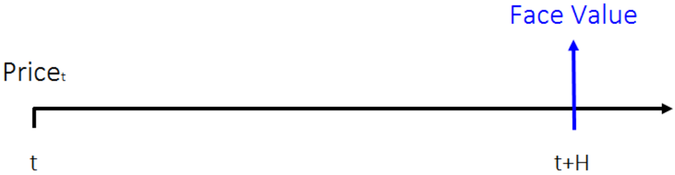
Module 4: Market Efficiency



Module 5: Debt Securities

$$P_t = \frac{c \cdot F}{(1+y)^1} + \frac{c \cdot F}{(1+y)^2} + \ldots + \frac{c \cdot F + F}{(1+y)^H}$$

This Section: (Default Free) Zero-Coupon Bonds



Valuation*

- Zero-Coupon Bond: borrow at time t and pay back at $t + H$

$$PV_t = \sum_{h=1}^{\infty} \frac{\mathbb{E}_t \left[CF_{t+h} \right]}{\left(1 + dr_{t,h}\right)^h}$$

$$\Downarrow$$

$$P_t = \frac{\mathbb{E}_t \left[Pay\ Back_{t+H} \right]}{\left(1 + dr_{t,H}\right)^H}$$

$$= \frac{Face\ Value}{\left(1 + dr_{t,H}\right)^H} \quad or \quad \frac{Par\ Value}{\left(1 + dr_{t,H}\right)^H}$$

$$= \frac{F}{\left(1 + dr_{t,H}\right)^H}$$

Explaining the Previous Slide

Zero-Coupon bonds have only one cash flow, which is paid at maturity. It is called the Face Value or the Par Value (we denote it by F). When we talk about zero coupon bonds we typically have in mind a bond with no default risk and, as such, F is received at maturity for sure. Based on these simple properties, we can use the valuation equation to find $P_t = \frac{F}{(1+dr_{t,H})^H}$ and this is true because $\mathbb{E}_t[CF_{t+h}] = 0$ for all years except at maturity, when $\mathbb{E}_t[CF_{t+h}] = F$.

Even though zero-coupon bonds with horizon higher than one year are typically not traded in financial markets, we can still think about a zero-coupon bonds of any maturity. This is useful because the price of any coupon paying bond will depend on the price of zero coupon bonds. As such, in order to understand coupon bonds we need to first understand zero-coupon bonds.

Yield to Maturity

- Market Participants often refer to "Yield to Maturity" (or "yield" for short). It is defined as:

$$P_t = \frac{F}{(1 + y_{t,H})^H} \qquad \times \qquad P_t = \frac{F}{(1 + dr_{t,H})^H}$$

- $y_{t,H} = dr_{t,H}$ (only for default free zero-coupon bonds).
- Lets call it y_H for simplicity, but y_H does vary over time.
- y_H is the per-period return when you hold the bond until maturity. Consider investing P_t at time t and holding it until maturity to receive F. Your total (gross) return is:

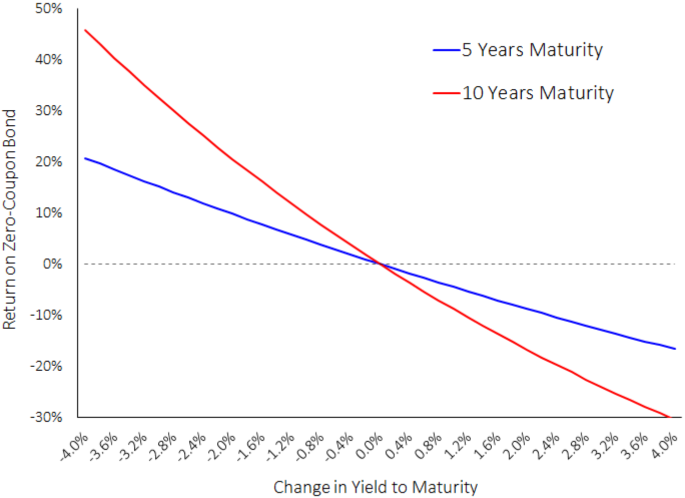
$$\begin{aligned} 1 + r &= \frac{F}{P_t} \\ &= (1 + y_H)^H \end{aligned}$$

Duration and Convexity Effects

$$P_t = \frac{F}{(1+y_H)^H}$$

- Prices are inversely related to yields (or interest rates):
  - $\uparrow y_H \quad \Rightarrow \quad \downarrow P_t$
  - $\downarrow y_H \quad \Rightarrow \quad \uparrow P_t$
- Duration, D, refers to the horizon of cash flows. With no coupons, the only cash flow is at maturity. Hence, $D = maturity$
- Bonds with higher D are more affected by movements in y_H (higher interest rate risk). This is called the "Duration Effect"
- Decreases in y_H induce a stronger effect than increases in y_H. This is called the "Convexity Effect" (and is desirable)

Duration and Convexity Effects*



Explaining the Previous Slide

Key message: Bond prices are inversely related to interest rates (they go up when interest rates go down and vice versa). This sensitivity to interest rates is stronger for bonds with longer horizon/maturity (i.e., higher duration bonds). Finally, the increase in prices when interest rates go down is generally larger than the decrease in prices when interest rates go up (this is the convexity effect).

Details:

The graph shows the return (y axis) on two zero-coupon bonds (a 5-year and a 10-year) when interest rates move. The initial interest rate (yield) is set to 8% and the x axis shows the change relative to this 8% value (4% means from 8% to 12%).

It is easy to see that for any given change in interest rates, the 10-year bond (which has a higher duration) has a return of larger magnitude than the 5-year bond. This means that higher duration bonds have higher sensitivity to interest rate movements (this is known as the duration effect). Intuitively, a cash flow to be paid in the distant future is discounted by a higher total discount rate (the annual yield is compounded to find the total discount rate), which means that higher duration bonds have prices that are more sensitive to movements in the annual interest rate.

Another observation from this graph is that the positive return when interest rates go up is larger (in magnitude) than the negative return when interest rates go down (this is the convexity effect). You can see this by comparing the return of almost 50% (-4% movement in yields) with the return of -30% (4% movement in yields).

Suppose you manage a portfolio of (synthetic) Zero-Coupon Bonds and you believe (contrary to the market) that interest rates are going down over the next months. If you are confident enough that you are right and the market is wrong, what should you do?

a) Tilt your position towards <u>longer-term bonds</u> since they have higher duration and, thus, <u>will provide higher positive return</u> if you are right

b) Tilt your position towards <u>shorter-term bonds</u> since they have higher duration and, thus, <u>will provide higher positive return</u> if you are right

c) Tilt your position towards <u>longer-term bonds</u> since they have higher duration and, thus, <u>will provide less negative return</u> if you are right

d) Tilt your position towards <u>shorter-term bonds</u> since they have higher duration and, thus, <u>will provide less negative return</u> if you are right

e) Nothing. All bonds of any maturity are exposed to movements in interest rates

The Yield Curve: Definition

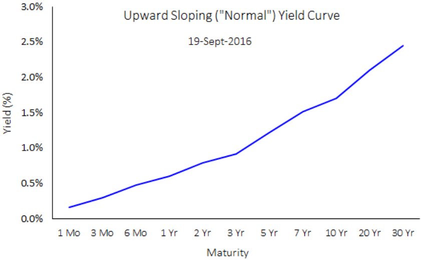
$$P_t = \frac{F}{(1+y_H)^H}$$

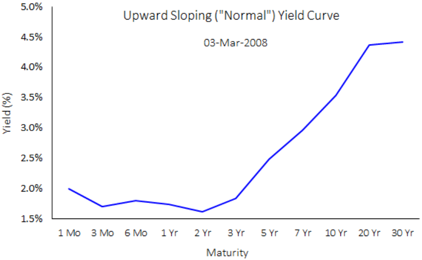
- Each maturity can have a different y_H and it is easy to invert the price formula to find y_H:

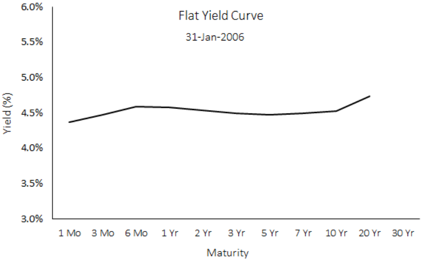
$$y_H = \left(\frac{F}{P_t}\right)^{1/H} - 1$$

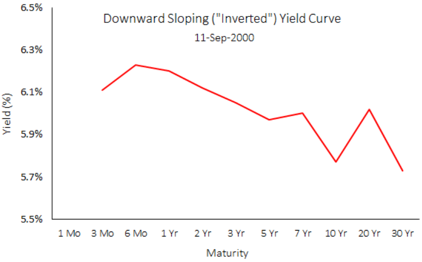
- The shape of the yield curve varies over time, but y_H typically increases in maturity ("upward sloping yield curve")
- Three main factors influence the shape of the yield curve:
  - Expectation of future yields
  - Liquidity (short-term bonds are more liquid)
  - Risk (the Duration effect induces long-term bonds to be riskier)

The Yield Curve: Alternative Shapes*









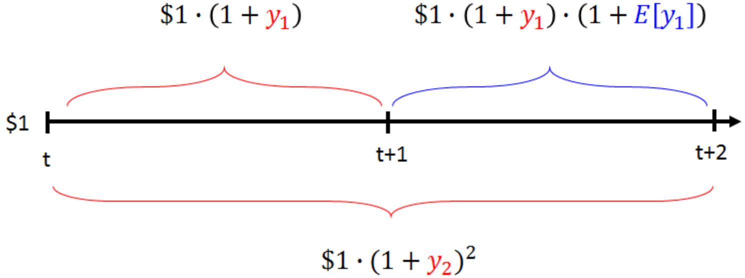
Source: www.treasury.gov

Explaining the Previous Slide

Each zero-coupon bond has a maturity, H, and each maturity has a respective yield to maturity, y_H. The yield curve simply plots y_H in the y-axis and the maturity in the x-axis. While the yield curve assumes different shapes over time, its typically upward sloping, which means that y_H increases with H. Since y_H is linked to expected returns, this means that longer-term bonds tend to have higher expected returns.

When we observe a yield curve that is downward sloping (as the graph in the bottom right), this typically means that markets expect yields to increase going forward. This can be made precise by the expectation hypothesis, which is outlined over the next slides.

The Yield Curve: The Expectation Hypothesis*



- If investors are indifferent between (i) investing in the longer-term bond and (ii) rolling over the shorter-term bond, then we have the "expectation hypothesis" equation:

$$(1 + y_2)^2 = (1 + y_1)(1 + \mathbb{E}[y_1]) \;\Rightarrow\; 1 + \mathbb{E}[y_1] = \frac{(1 + y_2)^2}{(1 + y_1)}$$

Explaining the Previous Slide

Consider the following two strategies:

- Buy 1$ of a 2-year zero-coupon bond and hold until maturity. This gives you $(1+y_2)^2$ at the end of the second year
- Buy 1$ of a 1-year zero-coupon bond to be held for one year and, at that point, invest the proceeds, $(1+y_1)$, into another 1-year zero-coupon bond to be held until maturity. This strategy (rolling over the shorter-term bonds) can be expected to give you $(1+y_1)(1+\mathbb{E}[y_1])$ at the end of the second year

If investors are indifferent between these two strategies, they must deliver the same expected final value and, thus, $(1+y_2)^2 = (1+y_1)(1+\mathbb{E}[y_1])$. This equation implies:

$$1+\mathbb{E}[y_1] = \frac{(1+y_2)^2}{(1+y_1)}$$

This is the "expectation hypothesis equation" and it allows us to interpret the yield curve as telling us something about expected interest rates going forward.

The Yield Curve: The Expectation Hypothesis

$$1 + \mathbb{E}[y_1] = \frac{(1 + y_2)^2}{(1 + y_1)}$$

- $\mathbb{E}[y_1]$ is the expectation for the future 1-year interest rate
- Using the expectation hypothesis equation, we can interpret the yield curve as telling us something about expected interest rates going forward
  - $y_2 = y_1 \quad \Rightarrow \quad \mathbb{E}[y_1] = y_1$
  - $y_2 > y_1 \quad \Rightarrow \quad \mathbb{E}[y_1] > y_1$
  - $y_2 < y_1 \quad \Rightarrow \quad \mathbb{E}[y_1] < y_1$
- If the expectation hypothesis were true, we would have alternating yield curves, but it would be flat on average. It is actually upward sloping on average. We need to account for risk and liquidity to understand that.

The Yield Curve: Liquidity and Risk Matter

$$y_2 - y_1 = (\mathbb{E}[y_1] - y_1) + \text{Liquidity Premium} + \text{Risk Premium}$$

- The Liquidity Premium
  - Shorter term treasury contracts are much more liquid
  - Liquidity Premium > 0
- The Risk Premium:
  - When investing in the longer-term bond, investors face higher interest rate risk (high duration)
  - When rolling over the shorter-term bond, investors face lower interest rate risk (low duration)
  - Risk Premium > 0 (alternative way to see the Duration Effect)

Regarding the shape of the yield curve:

a) It is most often downward sloping since shorter term bonds tend to have higher risk and lower liquidity

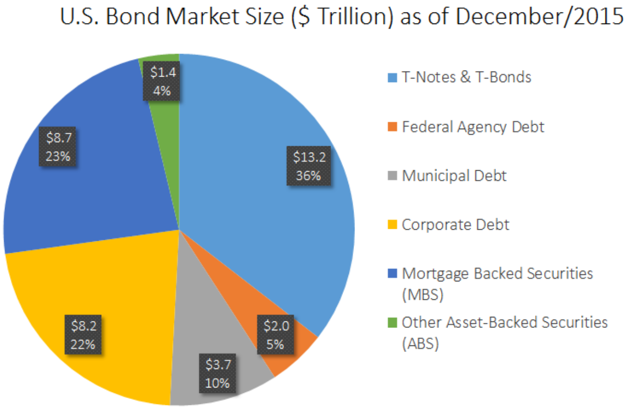
b) It is most often downward sloping since markets tend to expect interest rates to go down

c) It is most often upward sloping since longer term bonds tend to have higher risk and lower liquidity

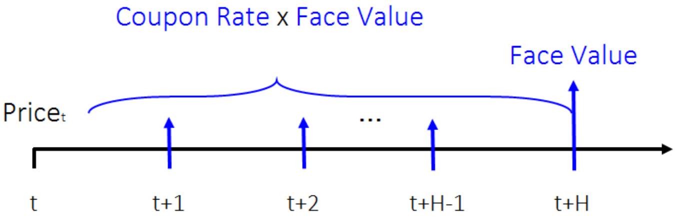
d) It is most often upward sloping since markets tend to expect interest rates to go up

e) It alternates, but it is, on average, flat since markets typically expect interest rates to remain at current level

This Section: Default free Debt (41% of the Market)



Cash Flows



Valuation*

- Bond with coupon rate of $c \cdot 100\%$ (interest paid annually) and maturity of H years:

$$PV_t = \sum_{h=1}^{\infty} \frac{\mathbb{E}_t\left[CF_{t+h}\right]}{\left(1+dr_{t,h}\right)^h}$$

$$\Downarrow$$

$$P_t = \frac{c \cdot F}{\left(1+dr_{t,1}\right)^1} + \frac{c \cdot F}{\left(1+dr_{t,2}\right)^2} + \ldots + \frac{c \cdot F + F}{\left(1+dr_{t,H}\right)^H}$$

$$= \frac{c \cdot F}{\left(1+y_1\right)^1} + \frac{c \cdot F}{\left(1+y_2\right)^2} + \ldots + \frac{c \cdot F + F}{\left(1+y_H\right)^H}$$

Explaining the Previous Slide

The valuation equation is given by $PV_t = \sum_{h=1}^{\infty} \frac{\mathbb{E}_t\left[CF_{t+h}\right]}{\left(1+dr_{t,h}\right)^h}$. To get bond prices we start by substituting expected cash flows by coupons, $c \cdot F$, at each year except for the last year since at that point we also receive the face value back (cash flow is $c \cdot F + F$). Using this we get $P_t = \frac{F}{\left(1+dr_{t,H}\right)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{\left(1+dr_{t,h}\right)^h}$

Since coupon bonds can be thought of as bundles of zero-coupon bonds, we know that the discount rate for maturity h is simply y_h (the yield on a zero-coupon bond with maturity h). Using this we get our final expression:

$$P_t = \frac{F}{(1+y_H)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y_h)^h}$$

One detail is that our pricing formula assumes annual interest payments and pricing at coupon date. Many bonds pay interest semi-annually and most trades happen between coupon dates. These facts add algebraic complexity, but do not change the logic of anything. The adjustment for these details can be found on the appendix of these class notes, but will not be tested.

Yield to Maturity

$$P_t = \frac{c \cdot F}{(1+y_1)^1} + \frac{c \cdot F}{(1+y_2)^2} + ... + \frac{c \cdot F + F}{(1+y_H)^H}$$

- We can still define the yield to maturity, y:

$$P_t = \frac{c \cdot F}{(1+y)^1} + \frac{c \cdot F}{(1+y)^2} + ... + \frac{c \cdot F + F}{(1+y)^H}$$

- The yield to maturity, y, depends on the entire yield curve (all y_h's), but it is just one number. It is selected to solve for the bond price (price is the same whether we use y or y_h's)

Yield to Maturity vs Coupon Rate

- The coupon rate, c, can be thought of as the rate at which cash flows are paid and the yield to maturity, y, as the rate at which cash flows are discounted
- They have an interesting relation:
  - If $y = c$, cash flows are paid and discounted at same rate and, thus, $P_t = F$ (the bond is "at par")
  - If $y > c$, cash flows are paid at a rate lower than they are discounted and, thus, $P_t < F$ (it is a "discount bond")
  - If $y < c$, cash flows are paid at a rate higher than they are discounted and, thus, $P_t > F$ (it is a "premium bond")
- Bonds are typically issued "at par", which means that the issuer selects the coupon rate investors currently require to impose no extra discount: $c = y$

Yield to Maturity vs Yield Curve

$$P_t = \frac{F}{(1+y_H)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y_h)^h} \quad \text{vs} \quad P_t = \frac{F}{(1+y)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y)^h}$$

- Depending on the analysis we will use the yield curve, y_h, or the yield to maturity of the bond, y.
- The advantage of using y is that it summarizes the information about bond "average return" in one number
  - If you hold the bond until maturity (and reinvest every coupon at y), then the total return on the bond is: $1 + r = (1 + y)^H$
  - If yields do not change from t to $t + 1$, then $r_{t+1} = y$
- Even if the y_h does not change, y will change as we get closer to maturity (decrease with an upward sloping yield curve). As such, the yield curve, y_h, allows us to better forecast our holding period returns (horizon analysis)

Duration

$$P_t = \frac{F}{(1+y)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y)^h}$$

- Recall that duration, D, refers to the horizon of cash flows. With no coupons, the only cash flow is at maturity and we have $D = H$. However, with coupons we use:

$$D = \sum_{h=1}^{H} w_h \cdot h \qquad \text{with} \qquad w_h = \begin{cases} \frac{c \cdot F}{(1+y)^h} / P_t & \text{for } h < H \\ \frac{c \cdot F + F}{(1+y)^h} / P_t & \text{for } h = H \end{cases}$$

- In words: Duration is the weighted average of the times cash flows are received with each weight equal to the contribution of the respective cash flow to the bond price

Modified Duration

- Duration is meant to capture the bond sensitivity to changes in y (bond risk). It turns that we can (linearly) approximate the return of the bond due to a change in the y as:

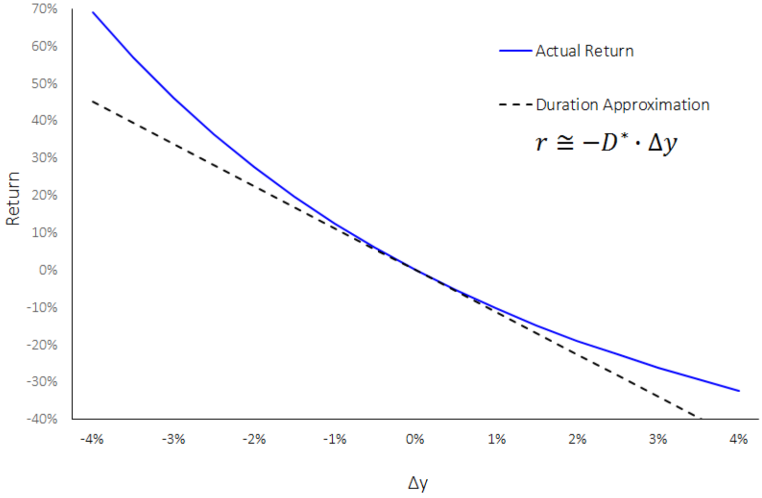
$$r \cong -\frac{D}{1+y} \cdot \Delta y$$

- This motivates us to define the "modified duration" as $D^* = \frac{D}{1+y}$ so that the previous equation simplifies to:

$$r \cong -D^* \cdot \Delta y$$

- This formula allows us to think about interest rate risk. D^* captures the sensitivity of bond returns to changes in the yield-to-maturity. For instance, with $D^* = 10$:
  - If yields increase by 1%, the bond return is approximately -10%
  - If yields decrease by 1%, the bond return is approximately 10%

Accuracy of Duration Approximation* (30-year 8% Coupon Bond at par)



Explaining the Previous Slide

Duration captures the interest rate risk of a bond. The way to see this it trough the approximation $r \cong -D^* \cdot \Delta y$. If (modified) duration is 10 and yields go up by 1%, then the bond goes down by approximately 10%.

How good is this approximation? Will the bond actually go down by 9.6% or by 6%? If the difference is large we are not properly measuring risk. The previous graph shows how the approximation works. It displays the duration approximation for a 30-year 8% coupon bond priced at par value ($P_t = F$).

When yield changes are small (e.g., $|\Delta y| < 1\%$) the approximation is pretty accurate (the blue line is close to the dashed line) and the return predicted by the duration formula is close to the actual return. However, when yield changes are large, the two lines become far apart and the duration approximation provides a prediction that is much lower than the actual return.

Over the short run, interest rates do not tend to change dramatically. That is why the duration approximation is so used in practice. Nevertheless, there is a way to improve upon that by using a quadratic approximation. This requires the calculation of bond convexity and will not be covered in this class.

Suppose the yield curve is upward sloping (yields are higher for longer-term bonds). If you buy a (default free) 10-year 8% coupon bond at par:

a) Its coupon rate is 8% at the purchasing date, but it will decrease over time if the yield curve does not change

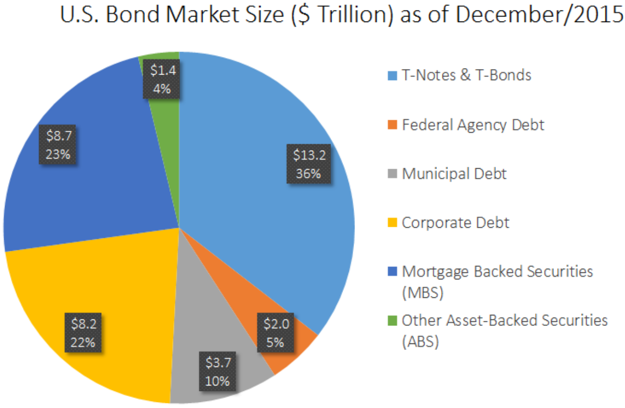
b) Its coupon rate is 8% at the purchasing date, but it will increase over time if the yield curve does not change

b) Its yield to maturity is 8% at the purchasing date, but it will decrease over time if the yield curve does not change

c) Its yield to maturity is 8% at the purchasing date, but it will increase over time if the yield curve does not change

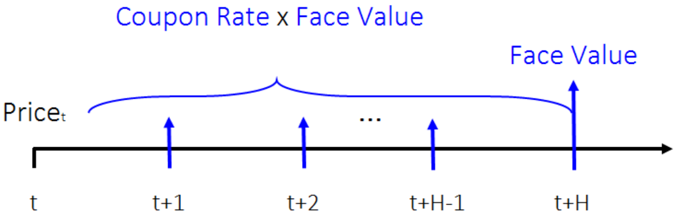
d) Its yield to maturity is 8% at the purchasing date and it will remain at 8% over time if the yield curve does not change

This Section: Debt with Default Risk (32% of the Market)



- Corporate Debt + Municipal Debt = 32% of market
- Asset-Backed Securities (including MBS) also face default risk. However, it would require an entire module to properly understand these (the basics were covered in the 2007-08 financial crisis class)

Cash Flows



You get these cash flows only if there is no default...

Bond Indentures/Covenants

- How do stockholders make sure CEO pays them dividends?
  - They have voting power
- Bondholders protect their rights using indentures/covenants:
  - Sinking Funds: to assure the company can pay the face value back, firms might agree to establish sinking funds, which are funds that repurchase bonds in the market before maturity
  - Serial Bonds: some bonds mature sequentially so that there is no accumulated cash being paid (downside is the illiquidity)
  - Subordination Clauses: restrict the amount of additional borrowing (typically requiring it to be subordinated)
  - Dividend Restrictions: limitations on dividend payments
  - Collateral: the firm can specify a particular asset that the bondholder receive if the firm defaults (called "collateral")

Credit Ratings and Historical Default Rates*

Company	Investment Grade Bonds		High Yield or Junk Bonds	
	Very High Quality	High Quality	Speculative	Very Poor
S&P	AAA & AA	A & BBB	BB & B	CCC & below
Moody's	Aaa & Aa	A & Baa	Ba & B	Caa & below

Moody's Rating	1 year	2 years	3 years	4 years	5 years	7 years	10 years
Aaa	0.00%	0.01%	0.01%	0.04%	0.11%	0.25%	0.50%
Aa	0.02%	0.06%	0.09%	0.16%	0.23%	0.38%	0.54%
A	0.05%	0.17%	0.34%	0.52%	0.72%	1.18%	2.05%
Baa	0.18%	0.50%	0.91%	1.40%	1.93%	3.00%	4.81%
Ba	1.17%	3.19%	5.58%	8.12%	10.40%	14.32%	19.96%
B	4.55%	10.43%	16.19%	21.26%	25.90%	34.47%	44.38%
Caa to C	17.72%	29.38%	38.68%	46.09%	52.29%	59.77%	71.38%

- Ex: If you buy a Baa rated corporate bond, there is a 3% probability that the firm will default within seven years from your purchase

Explaining the Previous Slide

The first table provides the general classification of defaultable bonds with two broad classes: "Investment Grade Bonds" and "High Yield Bonds" (or "Junk Bonds"). These two classes are further subdivided into categories based on ratings (provided by rating agencies such as Standard & Poor's (S&P) and Moody's). For instance, Aa is one possible (Moody's) rating for "Very High Quality Bonds".

The table does not show, but the rating agencies sometimes use extra adjustments to these ratings. In February 16, 2016 Moody's increased the rating of Ford long-term bonds from Baa3 to Baa2. The bonds were still Baa, but Baa2 is better than Baa3 (Baa1 is the strongest and Baa3 the weakest within Baa bonds). S&P does something similar, but it uses + and - (for instance, BBB+, BBB and BBB-).

While the actual notation used by the rating agencies differ, they are analogous. For instance, AAA and AA ratings from S&P are very high quality bonds just as Aaa and Aa from Moody's also are. A given bond may have ratings from multiple agencies and they often (roughly) agree, but it is not that uncommon to see two rating agencies giving different ratings to the same bond.

The second table provides some historical default rates based on Moody's rating categories (over period from 1970 to 2009). For instance, bonds rated B have generally defaulted 4.55% of the time within one year and 25.90% of the time within 5 years.

Valuation

- Bond with coupon rate of $c \cdot 100\%$ (interest paid annually) and maturity of H years:

$$P_t = \frac{F}{(1+y)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y)^h}$$

- Since there is a possibility for default, F and $c \cdot F$ represent the promised cash flows, but not the expected cash flows used in the fundamental valuation equation. The expectation needs to account for the possibility of default.

- As a result, the yield to maturity, y, is not equal to the overall discount rate, dr_t. This means that y does not measure the "average return" of the bond over its life. It actually measures the "promised average return", which is obtained if the bond does not default (maximum possible average return).

Yield to Maturity & Average Return

- Can we link y to the "average return"? Yes! If we assume:
  - Discount rates do not depend on horizon: $dr_{t,h} = dr_t$
  - Annual probability of default is p
  - Investors lose $\mathrm{LGD} \cdot 100\%$ of current bond price if the bond defaults ("loss given default", LGD, is typically relative to face value, but here I am using relative to bond price)
- Then we have that (proof in the appendix of these notes):

$$dr_t \quad \cong \quad y \quad - \quad \underbrace{p \cdot \mathrm{LGD}}_{\textit{adjustment for losses}}$$

  - if we are able to estimate p and LGD, we can use y to figure out dr_t (the average return of the bond over its life)
  - $y > dr_t$: we overestimate "average return" if we use only y
  - For high yield bonds $p \cong 5\%$ and $\mathrm{LGD} \cong 50\%$ and, thus, y is roughly 2.5% higher than the average return investors get

Yield to Maturity & Default Premium

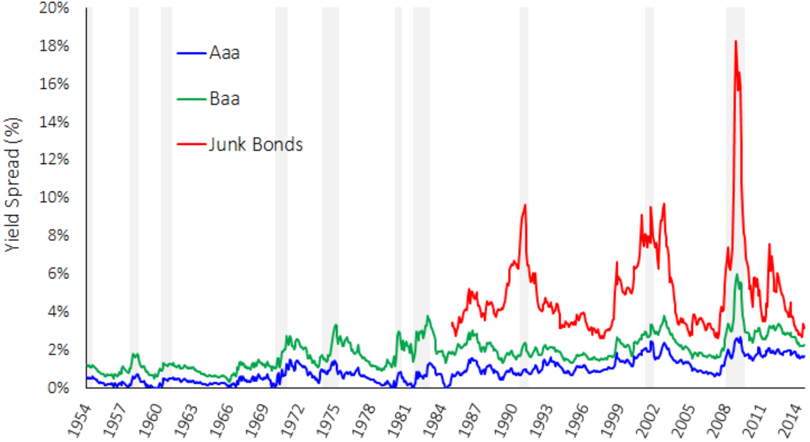
$$dr_t \cong y - p \cdot \mathrm{LGD}$$

- For a default free bond (i.e., $p = 0$), the yield to maturity already captures the "average return" of the bond (which is a result we saw in the previous sections)
- Let's call y^{df} the yield to maturity of a bond that is identical to our defaultable bond except that it is default free
- It is crucial to know how much more average return we get by facing default risk (labeled "effective default premium"):

$$\underbrace{dr_t - y^{df}}_{\textit{effective default premium}} \cong \underbrace{\left(y - y^{df}\right)}_{\textit{default premium}} - \underbrace{p \cdot \mathrm{LGD}}_{\textit{adjustment for losses}}$$

- Let's check an example in excel

Default Premium & Economic Recessions*



Explaining the Previous Slide

The graph shows the default spread (annual yield spread relative to treasury bonds with similar maturity) for portfolios of (i) Aaa rated bonds; (ii) Baa rated bonds and (iii) Junk Bonds (rated BB or lower). You can also see shaded regions in the graph. These represent periods of economic recession in the U.S. (as defined by the National Bureau of Economic Research). Two general patterns are important here:

- The yield spread is higher for bonds that are rated lower. This means that the market takes the ratings provided by rating agencies (in this case Moody's) very seriously and price lower rated bonds as having higher default risk
- The default premium varies systematically over the business cycle (for all bonds, but more so for junk bonds), with large increases during recessionary periods. Looking at our equation (isolating the yield spread now):

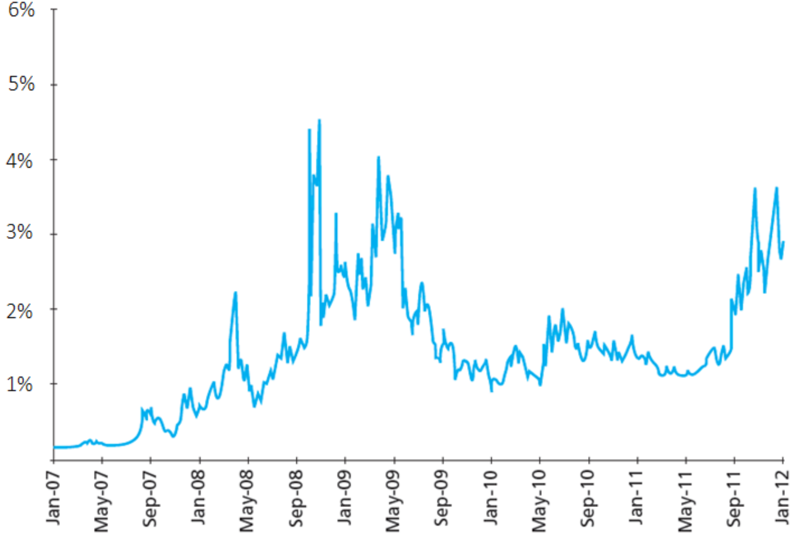
  $$y - y^{df} \cong \left(dr_t - y^{df}\right) + p \cdot \mathrm{LGD}$$

  we can see two drivers of the pattern observed: (i) investors believe probabilities of default (and maybe even the loss given default) to be higher during recessionary periods and (ii) investors require higher (effective) premium for holding riskier bonds during recessionary periods

Credit Default Swaps (CDS)

- CDS is an insurance policy on the default risk of a bond/loan
- Parties (buyer and seller) agree on
  - Reference Entity (the bond insured in this contract)
  - Notional amount (what the buyer gets if the credit event happens – typically a multiple of the face value of the bond)
  - Premium (% of notional amount paid annually by the buyer)
  - Length of contract (for how long the contract will remain)
- Ex: Buyer pays 1.3% of the face value of a Citigroup bond to the seller every year for 5 years. If Citigroup defaults (or decides to restructure its debt) at any point within these 5 years, the seller pays the face value of the bond to the buyer and receives the "bond in default" from the seller
- Buyer still faces default risk, but now both the bond issuer and the seller must default for the buyer to be affected

CDS Annual Premium (5-year contracts for US Banks)*



A: U.S. Banks

Explaining the Previous Slide

The graph shows the annual premium for CDS's of U.S. banks (averaged across banks) based on contracts with length of 5 years. The average premium is around 2%, which means that over this period you would need to pay around 2% (on average) of the face value of your bond if you wanted to insure it against default (and receive the face value back if a credit event happens).
Two points are important:

- The average CDS premium varies a lot over time. Close to Lehman Brothers failure (15-September-2008), the premium reached almost 5%, which is substantial for large banks
- A credit event does not necessarily means "default". It could mean a "restructuring of the debt" or other situation that goes against the bond holder (this must be specified in the contract agreement)

If a high yield corporate bond offers a yield to maturity of 12%, then:

a) The buyer can expect to receive an annual average return of 12% if he plans to hold the bond until maturity

b) The buyer can expect to receive an annual average return above 12% if he plans to hold the bond until maturity

c) The buyer can expect to receive an annual average return below 12% if he plans to hold the bond until maturity

d) The buyer will receive a return of 12% in each period he holds the bond

e) The buyer will receive a return of 12% in each period he holds the bond as long as the bond does not default

Appendix: Default Free Bond Valuation (Not Required)

- In section 3, I provide a formula for the valuation of a bond with coupon rate of $c \cdot 100\%$ (interest paid annually) and maturity of H years. Using yield to maturity, the formula is:

$$P_t = \frac{c \cdot F}{(1+y)^1} + \frac{c \cdot F}{(1+y)^2} + \frac{c \cdot F}{(1+y)^3} + \dots + \frac{c \cdot F + F}{(1+y)^H}$$

- If interest is paid k times per year instead ($k = 2$ for semi-annual payments), the formula becomes:

$$P_t = \frac{1/k \cdot c \cdot F}{(1+y)^{1/k}} + \frac{1/k \cdot c \cdot F}{(1+y)^{2 \cdot 1/k}} + \frac{1/k \cdot c \cdot F}{(1+y)^{3 \cdot 1/k}} + \dots + \frac{1/k \cdot c \cdot F + F}{(1+y)^H}$$

- Finally, if we are in between coupon payments (let's say n days for the next coupon), then the formula becomes:

$$P_t = \frac{1/k \cdot c \cdot F}{(1+y)^{n/365}} + \frac{1/k \cdot c \cdot F}{(1+y)^{n/365 + 1/k}} + \frac{1/k \cdot c \cdot F}{(1+y)^{n/365 + 2 \cdot 1/k}} + \dots + \frac{1/k \cdot c \cdot F + F}{(1+y)^H}$$

Appendix: Valuation of Defaultable Bonds (Not Required)

- At maturity, the bond price is equal to face value: $P_{t+H} = F$
- One year before maturity:

$$P_{t+H-1} = (1-p)\cdot\frac{P_{t+H} + c\cdot F}{1+dr_t} + p\cdot(1-\mathrm{LGD})\cdot P_{t+H-1}$$

$$= \frac{F + c\cdot F}{1+y} \qquad \textit{where} \quad 1+y = \frac{(1+dr_t)\cdot(1-p\cdot(1-\mathrm{LGD}))}{(1-p)}$$

- One year before that:

$$P_{t+H-2} = (1-p)\cdot\frac{(P_{t+H-1} + c\cdot F)}{1+dr_t} + p\cdot(1-\mathrm{LGD})\cdot P_{t+H-2}$$

$$= \frac{P_{t+H-1}}{1+y} + \frac{c\cdot F}{1+y}$$

$$= \frac{F + c\cdot F}{(1+y)^2} + \frac{c\cdot F}{1+y}$$

- We can keep doing this until reaching the current price of the bond (at time t):

$$P_t = \frac{F}{(1+y)^H} + \sum_{h=1}^{H}\frac{c\cdot F}{(1+y)^h} \qquad \textit{where} \quad 1+y = \frac{(1+dr_t)\cdot(1-p\cdot(1-\mathrm{LGD}))}{(1-p)}$$

- Taking log on both sides of the equation for y and using taylor expansion (from calculus) we get:

$$dr_t \cong y - p\cdot\mathrm{LGD}$$

References