Module 5: Debt Securities

(BUSFIN 4221 - Investments)

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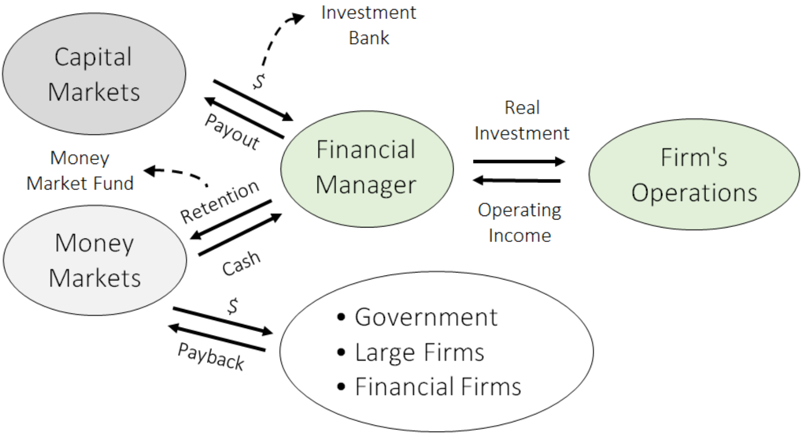
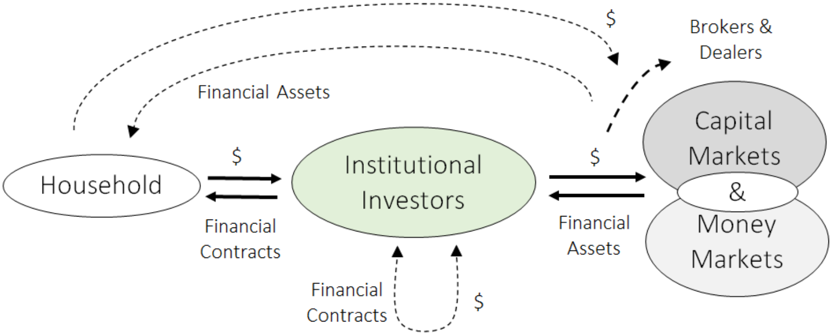
Fall 2016

Outline

Module 1 - The Demand for Capital



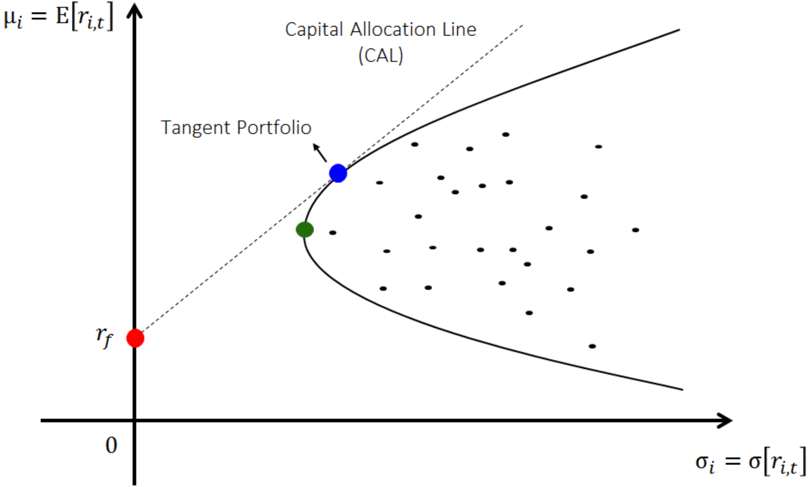
Module 1 - The Supply of Capital



Module 1 - Investment Principle

$$PV_t = \sum_{h=1}^{\infty} \frac{\mathbb{E}_t \left[CF_{t+h} \right]}{\left(1 + dr_{t,h}\right)^h}$$

Module 2 - Portfolio Theory



Module 3 - Factor Models

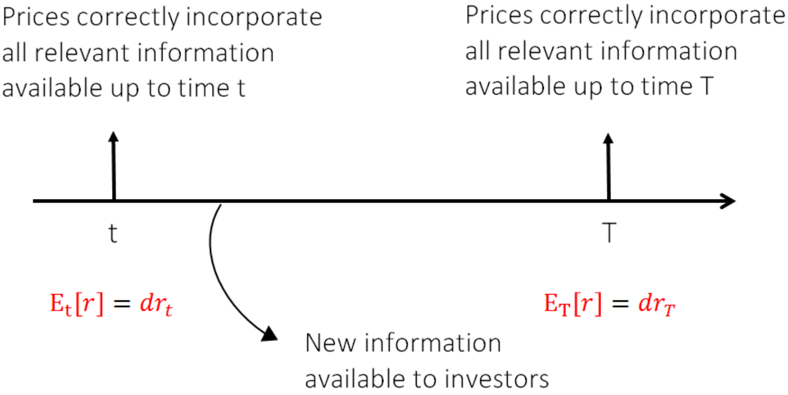
$$\mathbb{E}[r_i] = r_f + \beta_i \cdot (\mathbb{E}[r_M] - r_f)$$

$$+ \beta_{i,A} \cdot \mathbb{E}[r_A - r_a]$$

$$+ \beta_{i,B} \cdot \mathbb{E}[r_B - r_b]$$

$$+ \ldots$$

Module 4: Market Efficiency

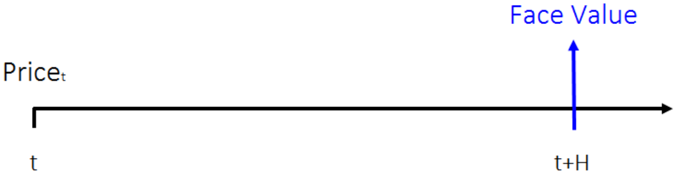


Module 5: Debt Securities

$$P_t = \frac{c \cdot F}{(1+y)^1} + \frac{c \cdot F}{(1+y)^2} + ... + \frac{c \cdot F + F}{(1+y)^H}$$

Outline

This Section: (Default Free) Zero-Coupon Bonds



Valuation*

- Zero-Coupon Bond: borrow at time t and pay back at $t + H$

$$PV_t = \sum_{h=1}^{\infty} \frac{\mathbb{E}_t \left[CF_{t+h} \right]}{\left(1 + dr_{t,h}\right)^h}$$

$$P_t = \frac{\mathbb{E}_t \left[Pay\ Back_{t+H} \right]}{\left(1 + dr_{t,H}\right)^H}$$

$$= \frac{Face\ Value}{\left(1 + dr_{t,H}\right)^H} \quad or \quad \frac{Par\ Value}{\left(1 + dr_{t,H}\right)^H}$$

$$= \frac{F}{\left(1 + dr_{t,H}\right)^H}$$

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Yield to Maturity

- Market Participants often refer to “Yield to Maturity” (or “yield” for short). It is defined as:

$$P_t = \frac{F}{(1 + y_{t,H})^H} \qquad \times \qquad P_t = \frac{F}{(1 + dr_{t,H})^H}$$

- $y_{t,H} = dr_{t,H}$ (only for default free zero-coupon bonds).
- Lets call it y_H for simplicity, but y_H does vary over time.
- y_H is the per-period return when you hold the bond until maturity. Consider investing P_t at time t and holding it until maturity to receive F. Your total (gross) return is:

$$1 + r = \frac{F}{P_t}$$
$$= (1 + y_H)^H$$

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Duration and Convexity Effects

$$P_t = \frac{F}{(1 + y_H)^H}$$

- Prices are inversely related to yields (or interest rates):
  - $\uparrow y_H \quad \Longrightarrow \quad \downarrow P_t$
  - $\downarrow y_H \quad \Longrightarrow \quad \uparrow P_t$
- Duration, D, refers to the horizon of cash flows. With no coupons, the only cash flow is at maturity. Hence, $D = maturity$
- Bonds with higher D are more affected by movements in y_H (higher interest rate risk). This is called the "Duration Effect"
- Decreases in y_H induce a stronger effect than increases in y_H. This is called the "Convexity Effect" (and is desirable)

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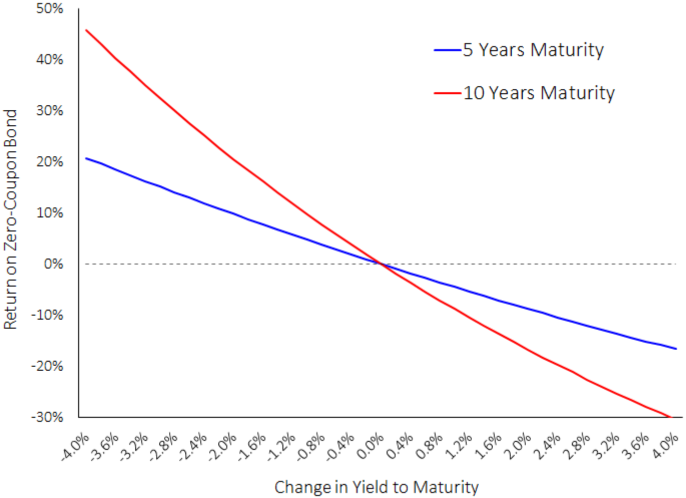

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a) Tilt your position towards longer-term bonds since they have higher duration and, thus, will provide higher positive return if you are right

b) Tilt your position towards shorter-term bonds since they have higher duration and, thus, will provide higher positive return if you are right

c) Tilt your position towards longer-term bonds since they have higher duration and, thus, will provide less negative return if you are right

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The Yield Curve: Definition

$$P_t = \frac{F}{(1+y_H)^H}$$

- Each maturity can have a different y_H and it is easy to invert the price formula to find y_H:

$$y_H = \left(\frac{F}{P_t}\right)^{1/H} - 1$$

- The shape of the yield curve varies over time, but y_H typically increases in maturity ("upward sloping yield curve")
- Three main factors influence the shape of the yield curve:
  - Expectation of future yields
  - Liquidity (short-term bonds are more liquid)
  - Risk (the Duration effect induces long-term bonds to be riskier)

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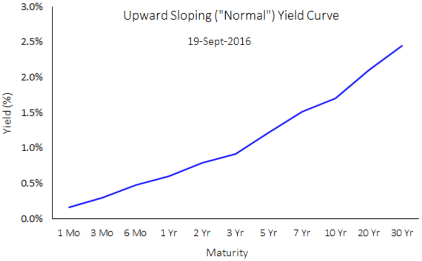

Source: www.treasury.gov

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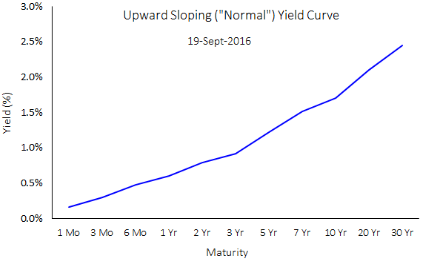

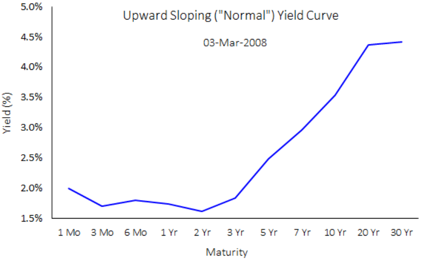

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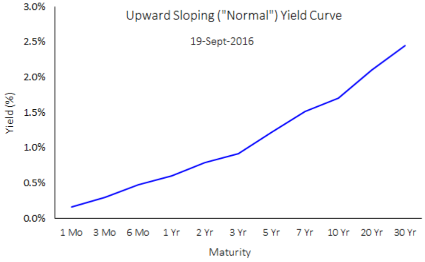

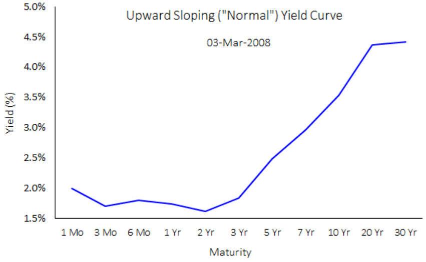

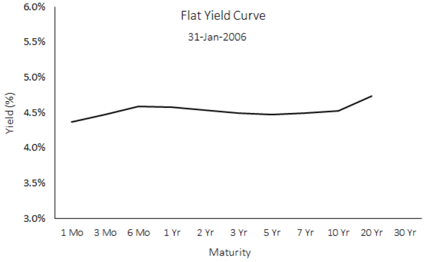

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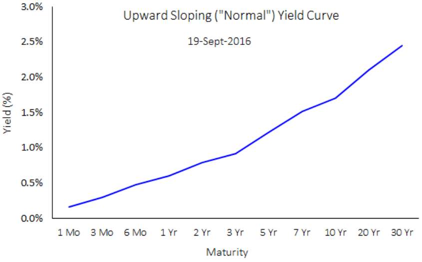

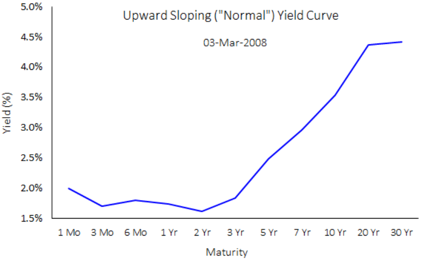

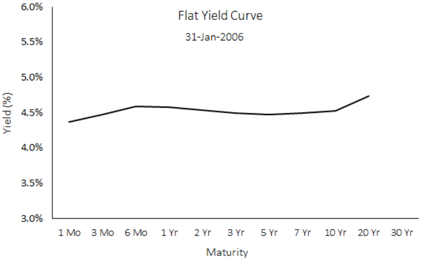

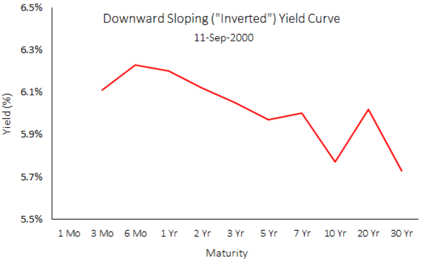

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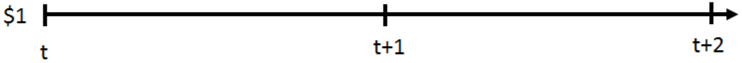
The Yield Curve: The Expectation Hypothesis*



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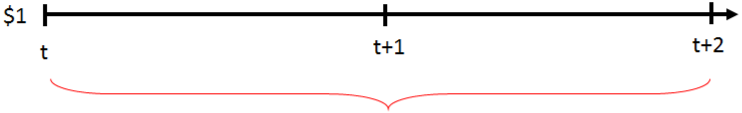

$$\$1 \cdot (1 + y_2)^2$$

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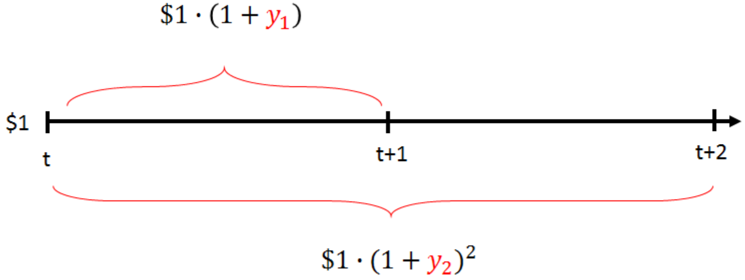

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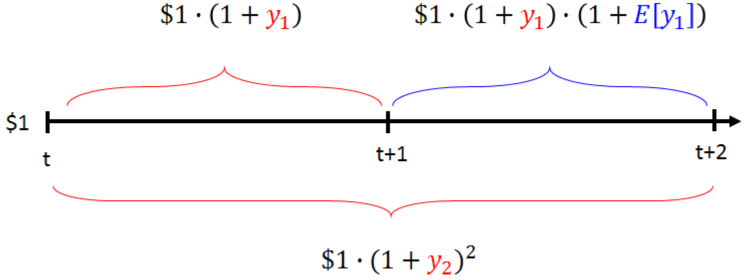

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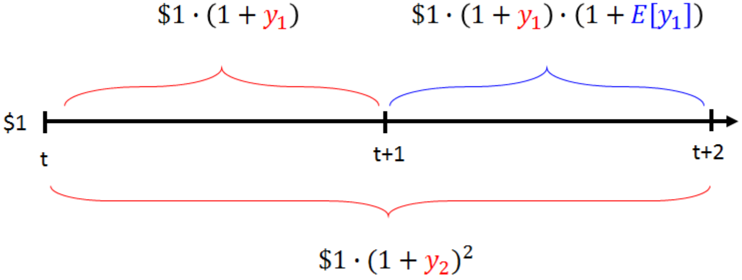

- If investors are indifferent between (i) investing in the longer-term bond and (ii) rolling over the shorter-term bond, then we have the "expectation hypothesis" equation:

$$(1+y_2)^2 = (1+y_1)\,(1+\mathbb{E}\,[y_1]) \;\Rightarrow\; 1+\mathbb{E}\,[y_1] = \frac{(1+y_2)^2}{(1+y_1)}$$

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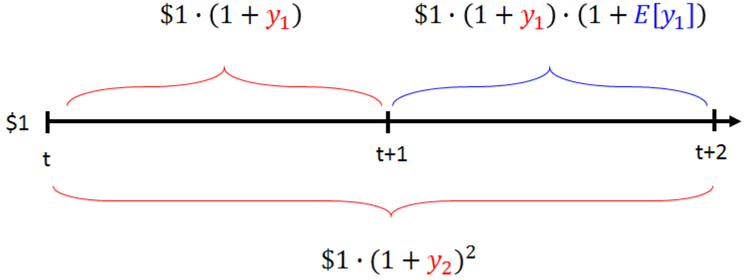

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The Yield Curve: The Expectation Hypothesis

$$1 + \mathbb{E}[y_1] = \frac{(1 + y_2)^2}{(1 + y_1)}$$

- $\mathbb{E}[y_1]$ is the expectation for the future 1-year interest rate
- Using the expectation hypothesis equation, we can interpret the yield curve as telling us something about expected interest rates going forward
  - $y_2 = y_1 \Rightarrow \mathbb{E}[y_1] = y_1$
  - $y_2 > y_1 \Rightarrow \mathbb{E}[y_1] > y_1$
  - $y_2 < y_1 \Rightarrow \mathbb{E}[y_1] < y_1$
- If the expectation hypothesis were true, we would have alternating yield curves, but it would be flat on average. It is actually upward sloping on average. We need to account for risk and liquidity to understand that.

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The Yield Curve: Liquidity and Risk Matter

$$y_2 - y_1 = (\mathbb{E}[y_1] - y_1) + \text{Liquidity Premium} + \text{Risk Premium}$$

- The Liquidity Premium
  - Shorter term treasury contracts are much more liquid
  - Liquidity Premium > 0
- The Risk Premium:
  - When investing in the longer-term bond, investors face higher interest rate risk (high duration)
  - When rolling over the shorter-term bond, investors face lower interest rate risk (low duration)
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Regarding the shape of the yield curve:

a) It is most often downward sloping since shorter term bonds tend to have higher risk and lower liquidity

b) It is most often downward sloping since markets tend to expect interest rates to go down

c) It is most often upward sloping since longer term bonds tend to have higher risk and lower liquidity

d) It is most often upward sloping since markets tend to expect interest rates to go up

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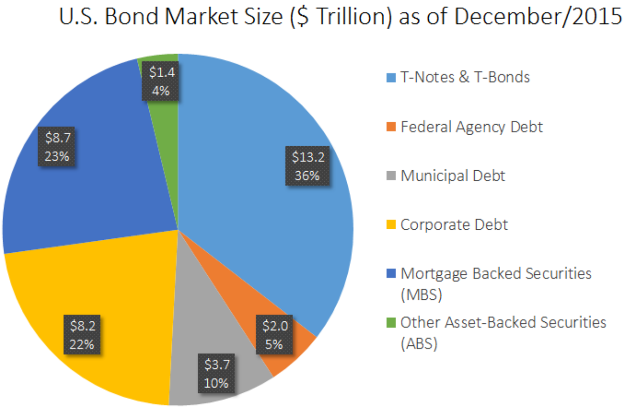
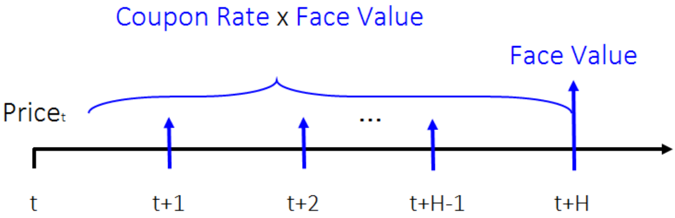
Outline

This Section: Default free Debt (41% of the Market)



Cash Flows



Valuation*

- Bond with coupon rate of $c \cdot 100\%$ (interest paid annually) and maturity of H years:

$$PV_t = \sum_{h=1}^{\infty} \frac{\mathbb{E}_t\left[CF_{t+h}\right]}{\left(1 + dr_{t,h}\right)^h}$$

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Yield to Maturity

$$P_t = \frac{c \cdot F}{(1+y_1)^1} + \frac{c \cdot F}{(1+y_2)^2} + ... + \frac{c \cdot F + F}{(1+y_H)^H}$$

- We can still define the yield to maturity, y:

$$P_t = \frac{c \cdot F}{(1+y)^1} + \frac{c \cdot F}{(1+y)^2} + ... + \frac{c \cdot F + F}{(1+y)^H}$$

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Yield to Maturity vs Coupon Rate

- The coupon rate, c, can be thought of as the rate at which cash flows are paid and the yield to maturity, y, as the rate at which cash flows are discounted
- They have an interesting relation:
  - If $y = c$, cash flows are paid and discounted at same rate and, thus, $P_t = F$ (the bond is "at par")
  - If $y > c$, cash flows are paid at a rate lower than they are discounted and, thus, $P_t < F$ (it is a "discount bond")
  - If $y < c$, cash flows are paid at a rate higher than they are discounted and, thus, $P_t > F$ (it is a "premium bond")
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Yield to Maturity vs Yield Curve

$$P_t = \frac{F}{(1+y_H)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y_h)^h} \quad \text{vs} \quad P_t = \frac{F}{(1+y)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y)^h}$$

- Depending on the analysis we will use the yield curve, y_h, or the yield to maturity of the bond, y.
- The advantage of using y is that it summarizes the information about bond "average return" in one number
  - If you hold the bond until maturity (and reinvest every coupon at y), then the total return on the bond is: $1 + r = (1+y)^H$
  - If yields do not change from t to $t+1$, then $r_{t+1} = y$
- Even if the y_h does not change, y will change as we get closer to maturity (decrease with an upward sloping yield curve). As such, the yield curve, y_h, allows us to better forecast our holding period returns (horizon analysis)

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$$P_t = \frac{F}{(1+y_H)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y_h)^h} \quad \text{vs} \quad P_t = \frac{F}{(1+y)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y)^h}$$

- Depending on the analysis we will use the yield curve, y_h, or the yield to maturity of the bond, y.
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$$D = \sum_{h=1}^{H} w_h \cdot h \qquad \text{with} \qquad w_h = \begin{cases} \frac{c \cdot F}{(1+y)^h} / P_t & \text{for } h < H \\ \frac{c \cdot F + F}{(1+y)^h} / P_t & \text{for } h = H \end{cases}$$

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Modified Duration

- Duration is meant to capture the bond sensitivity to changes in y (bond risk). It turns that we can (linearly) approximate the return of the bond due to a change in the y as:

$$r \cong -\frac{D}{1+y} \cdot \Delta y$$

- This motivates us to define the "modified duration" as $D^* = \frac{D}{1+y}$ so that the previous equation simplifies to:

$$r \cong -D^* \cdot \Delta y$$

- This formula allows us to think about interest rate risk. D^* captures the sensitivity of bond returns to changes in the yield-to-maturity. For instance, with $D^* = 10$:
  - If yields increase by 1%, the bond return is approximately -10%
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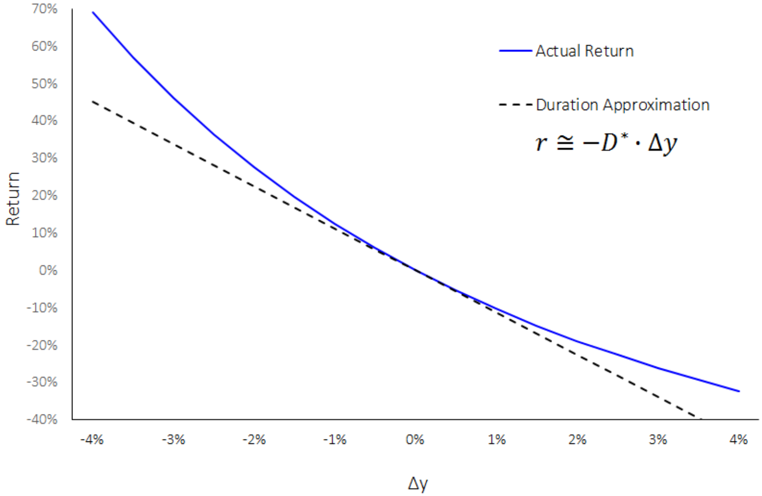
Accuracy of Duration Approximation*
(30-year 8% Coupon Bond at par)



Suppose the yield curve is upward sloping (yields are higher for longer-term bonds). If you buy a (default free) 10-year 8% coupon bond at par:

a) Its coupon rate is 8% at the purchasing date, but it will decrease over time if the yield curve does not change

b) Its coupon rate is 8% at the purchasing date, but it will increase over time if the yield curve does not change

c) Its yield to maturity is 8% at the purchasing date, but it will decrease over time if the yield curve does not change

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Outline

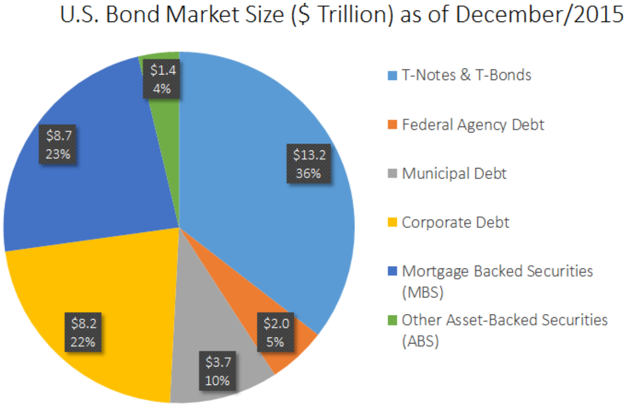
Overview

Zero-Coupon Bonds

Debt with no Default Risk

Debt with Default Risk

This Section: Debt with Default Risk (32% of the Market)



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- Asset-Backed Securities (including MBS) also face default risk. However, it would require an entire module to properly understand these (the basics were covered in the 2007-08 financial crisis class)

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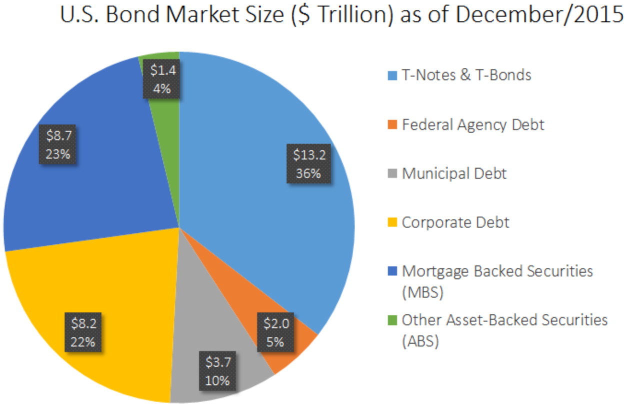

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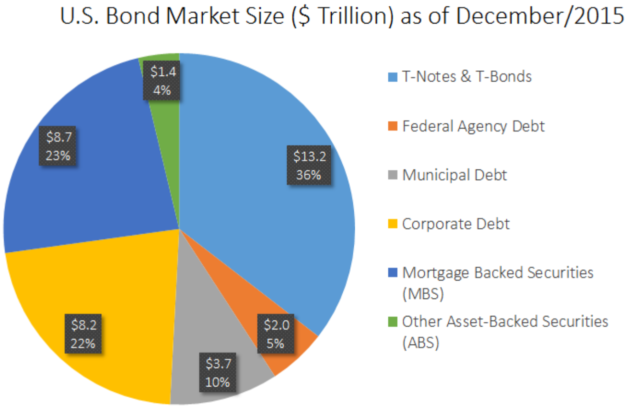

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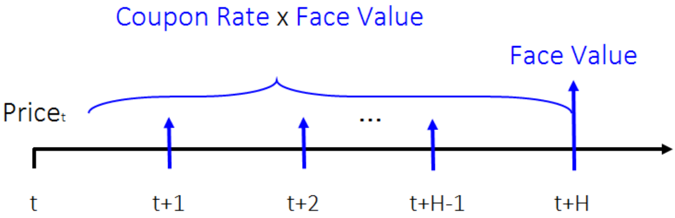
Cash Flows



You get these cash flows only if there is no default...

Bond Indentures/Covenants

- How do stockholders make sure CEO pays them dividends?
    - They have voting power
- Bondholders protect their rights using indentures/covenants:
    - Sinking Funds: to assure the company can pay the face value back, firms might agree to establish sinking funds, which are funds that repurchase bonds in the market before maturity
    - Serial Bonds: some bonds mature sequentially so that there is no accumulated cash being paid (downside is the illiquidity)
    - Subordination Clauses: restrict the amount of additional borrowing (typically requiring it to be subordinated)
    - Dividend Restrictions: limitations on dividend payments
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Credit Ratings and Historical Default Rates*

Company	Investment Grade Bonds		High Yield or Junk Bonds	
	Very High Quality	High Quality	Speculative	Very Poor
S&P	AAA & AA	A & BBB	BB & B	CCC & below
Moody's	Aaa & Aa	A & Baa	Ba & B	Caa & below

Moody's Rating	1 year	2 years	3 years	4 years	5 years	7 years	10 years
Aaa	0.00%	0.01%	0.01%	0.04%	0.11%	0.25%	0.50%
Aa	0.02%	0.06%	0.09%	0.16%	0.23%	0.38%	0.54%
A	0.05%	0.17%	0.34%	0.52%	0.72%	1.18%	2.05%
Baa	0.18%	0.50%	0.91%	1.40%	1.93%	3.00%	4.81%
Ba	1.17%	3.19%	5.58%	8.12%	10.40%	14.32%	19.96%
B	4.55%	10.43%	16.19%	21.26%	25.90%	34.47%	44.38%
Caa to C	17.72%	29.38%	38.68%	46.09%	52.29%	59.77%	71.38%

- Ex: If you buy a Baa rated corporate bond, there is a 3% probability that the firm will default within seven years from your purchase

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Valuation

- Bond with coupon rate of $c \cdot 100\%$ (interest paid annually) and maturity of H years:

$$P_t = \frac{F}{(1+y)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y)^h}$$

- Since there is a possibility for default, F and $c \cdot F$ represent the promised cash flows, but not the expected cash flows used in the fundamental valuation equation. The expectation needs to account for the possibility of default.

- As a result, the yield to maturity, y, is not equal to the overall discount rate, dr_t. This means that y does not measure the "average return" of the bond over its life. It actually measures the "promised average return", which is obtained if the bond does not default (maximum possible average return).

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- Can we link y to the "average return"? Yes! If we assume:
  - Discount rates do not depend on horizon: $dr_{t,h} = dr_t$
  - Annual probability of default is p
  - Investors lose $\mathrm{LGD} \cdot 100\%$ of current bond price if the bond defaults ("loss given default", LGD, is typically relative to face value, but here I am using relative to bond price)
- Then we have that (proof in the appendix of these notes):

$$dr_t \simeq y - \underbrace{\qquad}_{\text{adjustment for losses}}$$

  - if we are able to estimate and , we can use y to figure out dr_t (the average return of the bond over its life)
  - $y > dr_t$: we overestimate "average return" if we use only y
  - For high yield bonds $\simeq 5\%$ and $\simeq 50\%$ and, thus, y is roughly 2.5% higher than the average return investors get

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- Then we have that (proof in the appendix of these notes):

$$dr_t \quad \cong \quad y \quad - \quad \underbrace{p \cdot \mathrm{LGD}}_{\textit{adjustment for losses}}$$

  - if we are able to estimate p and LGD, we can use y to figure out dr_t (the average return of the bond over its life)
  - $y > dr_t$: we overestimate "average return" if we use only y
  - For high yield bonds $p \cong 5\%$ and $\mathrm{LGD} \cong 50\%$ and, thus, y is roughly 2.5% higher than the average return investors get

Yield to Maturity & Average Return

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Yield to Maturity & Default Premium

$$dr_t \cong y - p \cdot \mathrm{LGD}$$

- For a default free bond (i.e., $p = 0$), the yield to maturity already captures the "average return" of the bond (which is a result we saw in the previous sections)
- Let's call y^{df} the yield to maturity of a bond that is identical to our defaultable bond except that it is default free
- It is crucial to know how much more average return we get by facing default risk (labeled "effective default premium"):

$$\underbrace{dr_t - y^{df}}_{\text{effective default premium}} \cong \underbrace{\left(y - y^{df}\right)}_{\text{default premium}} - \underbrace{p \cdot \mathrm{LGD}}_{\text{adjustment for losses}}$$

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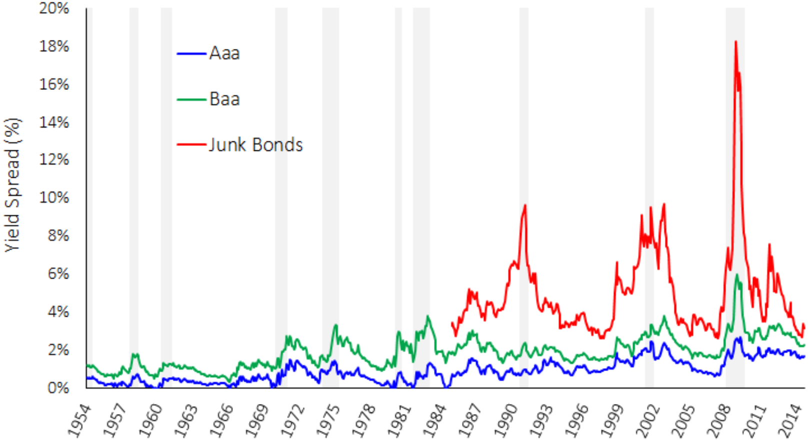
Default Premium & Economic Recessions*



Credit Default Swaps (CDS)

- CDS is an insurance policy on the default risk of a bond/loan
- Parties (buyer and seller) agree on
  - Reference Entity (the bond insured in the contract)
  - Notional amount (what the buyer gets if the credit event happens – typically a multiple of the face value of the bond)
  - Premium (% of notional amount paid annually by the buyer)
  - Length of contract (for how long the contract will remain)
- Ex: Buyer pays 1.3% of the face value of a Citigroup bond to the seller every year for 5 years. If Citigroup defaults (or decides to restructure its debt) at any point within these 5 years, the seller pays the face value of the bond to the buyer and receives the "bond in default" from the seller
- Buyer still faces default risk, but now both the bond issuer and the seller must default for the buyer to be affected

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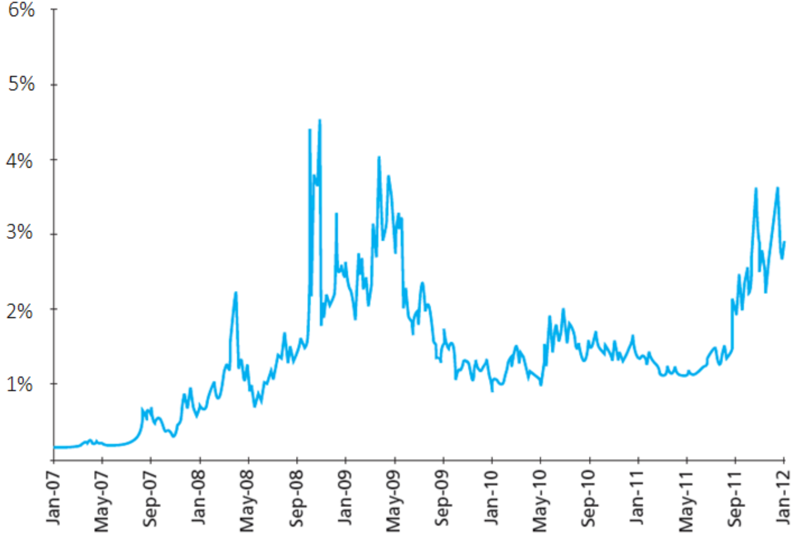
CDS Annual Premium (5-year contracts for US Banks)*



A: U.S. Banks

If a high yield corporate bond offers a yield to maturity of 12%, then:

a) The buyer can expect to receive an annual average return of 12% if he plans to hold the bond until maturity

b) The buyer can expect to receive an annual average return above 12% if he plans to hold the bond until maturity

c) The buyer can expect to receive an annual average return below 12% if he plans to hold the bond until maturity

d) The buyer will receive a return of 12% in each period he holds the bond

e) The buyer will receive a return of 12% in each period he holds the bond as long as the bond does not default

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Appendix: Default Free Bond Valuation (Not Required)

- In section 3, I provide a formula for the valuation of a bond with coupon rate of $c \cdot 100\%$ (interest paid annually) and maturity of H years. Using yield to maturity, the formula is:

$$P_t = \frac{c \cdot F}{(1+y)^1} + \frac{c \cdot F}{(1+y)^2} + \frac{c \cdot F}{(1+y)^3} + \ldots + \frac{c \cdot F + F}{(1+y)^H}$$

- If interest is paid k times per year instead ($k = 2$ for semi-annual payments), the formula becomes:

$$P_t = \frac{1/k \cdot c \cdot F}{(1+y)^{1/k}} + \frac{1/k \cdot c \cdot F}{(1+y)^{2 \cdot 1/k}} + \frac{1/k \cdot c \cdot F}{(1+y)^{3 \cdot 1/k}} + \ldots + \frac{1/k \cdot c \cdot F + F}{(1+y)^H}$$

- Finally, if we are in between coupon payments (let's say n days for the next coupon), then the formula becomes:

$$P_t = \frac{1/k \cdot c \cdot F}{(1+y)^{n/365}} + \frac{1/k \cdot c \cdot F}{(1+y)^{n/365 + 1/k}} + \frac{1/k \cdot c \cdot F}{(1+y)^{n/365 + 2 \cdot 1/k}} + \ldots + \frac{1/k \cdot c \cdot F + F}{(1+y)^H}$$

Appendix: Valuation of Defaultable Bonds (Not Required)

- At maturity, the bond price is equal to face value: $P_{t+H} = F$
- One year before maturity:

$$P_{t+H-1} = (1-p) \cdot \frac{P_{t+H} + c \cdot F}{1 + dr_t} + p \cdot (1 - \mathrm{LGD}) \cdot P_{t+H-1}$$

$$= \frac{F + c \cdot F}{1+y} \qquad \textit{where} \quad 1 + y = \frac{(1+dr_t) \cdot (1 - p \cdot (1 - \mathrm{LGD}))}{(1-p)}$$

- One year before that:

$$P_{t+H-2} = (1-p) \cdot \frac{(P_{t+H-1} + c \cdot F)}{1 + dr_t} + p \cdot (1 - \mathrm{LGD}) \cdot P_{t+H-2}$$

$$= \frac{P_{t+H-1}}{1+y} + \frac{c \cdot F}{1+y}$$

$$= \frac{F + c \cdot F}{(1+y)^2} + \frac{c \cdot F}{1+y}$$

- We can keep doing this until reaching the current price of the bond (at time t):

$$P_t = \frac{F}{(1+y)^H} + \sum_{h=1}^{H} \frac{c \cdot F}{(1+y)^h} \qquad \textit{where} \quad 1 + y = \frac{(1+dr_t) \cdot (1 - p \cdot (1 - \mathrm{LGD}))}{(1-p)}$$

- Taking log on both sides of the equation for y and using taylor expansion (from calculus) we get:

$$dr_t \cong y - p \cdot \mathrm{LGD}$$