Fundamental Anomalies

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## Outline

The Paper

My Comments

Final Remarks
$\mathbb{E}[R]$ from Firm's Perspective

## $\mathbb{E}[R]$ from Firm's Perspective

- Firm's Net Present Value (NPV) Rule:

$$
\mathrm{NPV}=\quad \text { Inv } \partial \text { Benefit } \quad-\quad \text { Inv } \partial \text { Cost }
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\begin{array}{rlll}
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& =\widehat{C F}(I) /(1+W A C C) & - & \operatorname{Cost}(I)
\end{array}
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0 & =\mathbb{E}\left[C F\left(I^{*}\right)\right] / \mathbb{E}[R] & & -\operatorname{Cost}\left(I^{*}\right)
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& & \\
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q-theory formalizes this logic and implies: $R^{S}=R^{F}$

The Empirical Challenge of $\mathbb{E}\left[R^{S}\right]=\mathbb{E}\left[R^{F}\right]$

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|  | $\boxed{\mathrm{SUE}}$ | $\mathrm{B} / \mathrm{M}$ | $\overline{\mathrm{CI}}$ |
| :---: | :---: | :---: | :---: |
| Matching |  |  |  |
|  | Expected | Returns |  |
| $a$ | 7.7 | 22.3 | 1.0 |
|  | $[1.7]$ | $[25.5]$ | $[.3]$ |
| $\alpha$ | .3 | .5 | .2 |
|  | $[.0]$ | $[.3]$ | $[.0]$ |



## Campbell (2017): The Empirical Challenge Matters


"This problem, that different parameters are needed to fit each anomaly, is a pervasive one in the $q$-theoretic asset pricing literature (p. 275)."

Gonçalves, Xue, and Zhang (2020, RFS)

$$
r_{p}^{F}=r^{F}\left(r_{p}^{B a} ; w_{p}^{B} ; \frac{I_{p}}{K_{p}} ; \frac{Y_{p}}{K_{p}} ; \delta_{p}\right)
$$

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\begin{aligned}
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& r_{p}^{F}=\sum_{i=1}^{N_{p}} w_{i p} \times r^{F}\left(r_{i}^{B a} ; w_{i}^{B} ; \frac{I_{i}}{K_{i}} ; \frac{I_{i}}{K_{i}} ; \frac{Y_{i}}{K_{i}+W_{i}} ; \frac{K_{i}}{K_{i}+W_{i}} ; \delta_{i}\right)
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\end{aligned}
$$

Equal-Weighted Portfolios


Value-Weighted Portfolios


## This Paper

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$$
r_{i}^{F}=r^{F}\left(r_{i}^{B a} ; w_{i}^{B} ; \frac{l_{i}}{K_{i}} ; \frac{l_{i}}{K_{i}} ; \frac{Y_{i}}{K_{i}+W_{i}} ; \frac{K_{i}}{K_{i}+W_{i}} ; \delta_{i}\right)
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Consumer Nondurables: $\gamma$


Consumer Nondurables: a


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Business Equipment: $\gamma$


Consumer Nondurables: a


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$\mathbb{E}[r]$ for 12 Anomalies


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## Aggregate Returns



## Outline

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## 1) Compare Bayesian and Frequentist Methods

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r_{i t+1}^{S}=r_{i t+1}^{F}+\varpi_{i t}^{-1 / 2} \cdot \sigma_{r} \cdot e_{i t+1}^{r} \quad \text { with } e_{i t}^{r} \stackrel{i i d}{\sim} N(0,1)
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- You can estimate $\theta_{j t}$ at each $t$ and for each $j$ (by NLS)
- How much does the fit improve as you change from the Frequentist (NLS) to the Bayesian (MCMC) framework?

2) Check if Variation in $\gamma_{j t}$ and $a_{j t}$ is Economically Sensible

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- $\left(V_{i t}+B_{i t+1}\right) / K_{i t}=Q_{i t}=\left(1+a_{j t} \cdot\left(1-\tau_{t}\right) \cdot I_{i t} / K_{i t}\right)+W_{i t} / K_{i t}$

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- Is variation in $a_{j t}$ in line with variation in $Q_{j t}$ ?

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r_{r_{t+1}^{s}}^{s}=r_{i t+1}^{F}+w_{i t}^{-1 / 2} \cdot \sigma_{r} \cdot e_{i t+1}^{r} \quad \text { with } e_{i t}^{r} \sim i^{i d} d(0,1)
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- Is variation in $a_{j t}$ in line with variation in $Q_{j t}$ ?
- Is variation in $a_{j t}$ in line with the sensitivity of $Q_{i t}$ to $I_{i t} / K_{i t}$ ?

3) Deal with Misalignment Between $r_{i t}^{S}$ and $r_{i t}^{F}$

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r_{i t+1}^{S}=r_{i t+1}^{F}+\varpi_{i t}^{-1 / 2} \cdot \sigma_{r} \cdot e_{i t+1}^{r} \quad \text { with } e_{i t}^{r} \stackrel{i i d}{\sim} N(0,1)
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$$

- Impossible to perfectly align $r_{i t}^{S}$ and $r_{i t}^{F}$

3) Deal with Misalignment Between $r_{i t}^{S}$ and $r_{i t}^{F}$

$$
r_{r_{t+1}^{s}}^{s}=r_{i t+1}^{E}+\varpi_{i t}^{-1 / 2} \cdot \sigma_{r} \cdot e_{t+1}^{r} \quad \text { with } e_{i t}^{r} \sim N(0,1)
$$

- Impossible to perfectly align $r_{i t}^{S}$ and $r_{i t}^{F}$
- Not relevant for prior papers as they rely on (portfolio) $\mathbb{E}[r]$

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- Overfitting: time-varying $\theta_{j t}$ compensates for misalignment

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- Very relevant for this paper
- Overfitting: time-varying $\theta_{j t}$ compensates for misalignment
- This is why the specification with $\theta_{j}$ performs better OOS

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- MA parameters assure $r_{i t}^{S}-r_{i t}^{F}$ is uncorrelated

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- MA parameters assure $r_{i t}^{S}-r_{i t}^{F}$ is uncorrelated
- Recover $r_{i t}^{F}$ and use $r_{i t+1}^{S}=r_{i t}^{F}+\varpi_{i t}^{-1 / 2} \cdot \sigma_{r} \cdot e_{i t+1}^{r}$

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## My Comments

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- Good luck!

