International Arbitrage Premia

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## Outline

The Paper

My Comments

Final Remarks

## The Paper in a Nutshell

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- Linear SDF: $M_{t+1}^{\star}=a_{t}+b_{t} \cdot r_{t+1}$


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\begin{aligned}
& \text { Minimize } \mathbb{E}_{t}\left[M_{t+1}^{2}\right] \\
& \text { s.t. } \quad \mathbb{E}_{t}\left[M_{t+1}\right]=1 / R_{f, t+1} \\
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- Study the Residual Mispricing: $R M P_{t+1}=M_{t+1}^{o}-\mathbb{E}_{t}\left[M_{t+1}^{o 2}\right]$ where $M_{t}^{o}=M_{t}-M_{t}^{\star}$

Figure 1: Time series of international residual mispricing


Table 1: Determinants of RMP

| Panel A: U.S. Residual Mispricing |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Financial uncertainty | $\begin{gathered} 0.441^{\star \star \star} \\ {[0.086]} \end{gathered}$ |  |  |  | $\begin{gathered} 0.392^{\star \star \star} \\ {[0.063]} \end{gathered}$ |  |
| VIX |  | $\begin{gathered} 0.500^{\star \star \star} \\ {[0.112]} \end{gathered}$ |  |  |  | $\begin{gathered} 0.490^{\star \star \star} \\ {[0.103]} \end{gathered}$ |
| Intermediary leverage |  |  | $\begin{gathered} 0.234^{\star \star \star} \\ {[0.073]} \end{gathered}$ |  | $\begin{gathered} 0.056 \\ {[0.049]} \end{gathered}$ | $\begin{gathered} -0.006 \\ {[0.058]} \end{gathered}$ |
| TED spread |  |  |  | $\begin{aligned} & 0.258^{\star \star} \\ & {[0.127]} \end{aligned}$ | $\begin{gathered} 0.132 \\ {[0.095]} \end{gathered}$ | $\begin{gathered} 0.082 \\ {[0.080]} \end{gathered}$ |
| $R^{2}$ | 19.2\% | 24.9\% | 5.3\% | 6.5\% | 21.8\% | 26.8\% |
| Panel B: International Residual Mispricing |  |  |  |  |  |  |
| Financial uncertainty | $\begin{gathered} 0.445^{\star \star \star} \\ {[0.095]} \end{gathered}$ |  |  |  | $\begin{gathered} 0.396^{\star \star \star} \\ {[0.069]} \end{gathered}$ |  |
| VIX |  | $\begin{gathered} 0.506^{\star \star \star} \\ {[0.138]} \end{gathered}$ |  |  |  | $\begin{gathered} 0.494^{\star \star \star} \\ {[0.126]} \end{gathered}$ |
| Intermediary leverage |  |  | $\begin{gathered} 0.206^{\star \star \star} \\ {[0.072]} \end{gathered}$ |  | $\begin{gathered} 0.016 \\ {[0.050]} \end{gathered}$ | $\begin{gathered} -0.047 \\ {[0.062]} \end{gathered}$ |
| TED spread |  |  |  | $\begin{aligned} & 0.289^{\star \star} \\ & {[0.140]} \end{aligned}$ | $\begin{gathered} 0.173 \\ {[0.112]} \end{gathered}$ | $\begin{gathered} 0.123 \\ {[0.090]} \end{gathered}$ |
| $R^{2}$ | 19.6\% | 25.4\% | 4.0\% | 8.2\% | 22.6\% | 27.7\% |

Table 1: Determinants of RMP


Table 5: RMP and statistical arbitrage in the data
Panel A: Contemporaneous regressions

|  | U.S. Residual Mispricing |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | MDI | CIP 3M | CIP 1Y | CIP 2Y | CIP 3Y | CIP 5Y | CIP 7Y | CIP 10Y |
| $\beta$ | $0.331^{*}$ | $0.320^{*}$ | $0.302{ }^{\text {** }}$ | 0.361 ** | $0.348^{\star \star}$ | $0.331^{\star \star \star}$ | $0.255^{\star \star \star}$ | $0.190^{\star \star \star}$ |
|  | [0.171] | [0.195] | [0.132] | [0.145] | [0.141] | [0.121] | [0.099] | [0.069] |
| $R^{2}$ | 10.7\% | 9.8\% | 8.7\% | 12.7\% | 11.8\% | 10.6\% | 6.1\% | 3.2\% |
| International Residual Mispricing |  |  |  |  |  |  |  |  |
|  | MDI | CIP 3M | CIP 1Y | CIP 2 Y | CIP 3Y | CIP 5Y | CIP 7Y | CIP 10Y |
| $\beta$ | 0.285* | 0.267 | 0.245* | $0.318^{* *}$ | $0.319^{* *}$ | $0.296{ }^{* *}$ | $0.222^{\text {** }}$ | $0.178^{* * *}$ |
|  | [0.148] | [0.201] | [0.143] | [0.150] | [0.141] | [0.123] | [0.098] | [0.069] |
| $R^{2}$ | 7.8\% | 6.8\% | 5.6\% | 9.8\% | 9.8\% | 8.4\% | 4.5\% | 2.8\% |

Panel B: Predictive regressions

|  | U.S. Residual Mispricing |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | MDI | CIP 3M | CIP 1Y | CIP 2Y | CIP 3Y | CIP 5Y | CIP 7Y | CIP 10Y |
| $\beta$ | $0.301^{\star \star}$ | 0.231 | $0.350^{\star \star}$ | $0.399^{\star \star \star}$ | $0.384^{\star \star}$ | $0.368^{\star \star \star}$ | $0.279^{\star \star \star}$ | $0.210^{\star \star \star}$ |
|  | $[0.139]$ | $[0.173]$ | $[0.152]$ | $[0.161]$ | $[0.152]$ | $[0.129]$ | $[0.098]$ | $[0.067]$ |
| $R^{2}$ | $8.7 \%$ | $5.0 \%$ | $11.9 \%$ | $15.5 \%$ | $14.4 \%$ | $13.2 \%$ | $7.4 \%$ | $4.0 \%$ |

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\beta$ | $0.275^{\star \star}$ | 0.213 | $0.283^{\star}$ | $0.349^{\star \star}$ | $0.350^{\star \star}$ | $0.329^{\star \star}$ | $0.237^{\star \star}$ | $0.197^{\star \star \star}$ |
|  | $[0.130]$ | $[0.172]$ | $[0.161]$ | $[0.167]$ | $[0.156]$ | $[0.131]$ | $[0.099]$ | $[0.067]$ |
| $R^{2}$ | $7.3 \%$ | $4.1 \%$ | $7.6 \%$ | $11.8 \%$ | $11.9 \%$ | $10.5 \%$ | $5.2 \%$ | $3.5 \%$ |

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Table 7: RMP index and asset returns

|  | Equity | Currency |
| :--- | :---: | :---: |
| $\lambda$ | $0.035^{\star \star \star}$ | $0.040^{\star \star \star}$ |
|  | $(5.233)$ | $(5.077)$ |
| RMSE | 0.0008 | 0.0003 |
| $R^{2}$ | $40.8 \%$ | $90.1 \%$ |

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Final Remarks

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- We could have $M_{t}=M_{t}^{\star}+M_{t}^{o}>0$ with $M_{t}^{o} \perp \mathbb{R}_{t}$
- So, whether $M_{t}^{\star}<0$ implies arbitrage opportunities depends on whether $M_{t}^{o}$ prices other assets or not
- This discussion needs to be present in the text (+ it motivates testing whether $M_{t}^{o}$ prices assets beyond $M_{t}^{\star}$ )

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- As such, I would expect $M_{t}^{o}>0$ in good times
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- What drives that? $\left(b_{t}\right.$ ? $b<0$ ? ... $)$

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- But in this case a clean discussion of the economics behind this analysis is needed


## Other Comments

1. The interpretation of $R M P_{t}$ as the profit associated with an insurance strategy implicitly assumes that $M_{t}^{o}$ is a tradable payoff. I suggest you add a discussion of this aspect
2. $D / P$ is a typical state variable predicting the equity premium. I suggest you add $D / P$ to the set of state variables used to estimate $\mathbb{E}_{t}[\cdot]$
3. Typical models induce an exponential SDF. I suggest you provide a more detailed discussion about why a polynomial SDF is preferred over an exponential SDF

## Outline

## The Paper

## My Comments

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- Good luck!

