THE OHIO STATE UNIVERSITY
FISHER COLLEGE OF BUSINESS

The Subjective Belief Factor

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Discussant: **Andrei S. Gonçalves**

2026 U of Toronto Asset Pricing & Investments Workshop

Outline

The Paper

My Comments

Final Remarks

The Big Picture Motivation

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- Insight: use asset pricing methods to fully characterize $\mathbb{E}_t^*[\cdot]$
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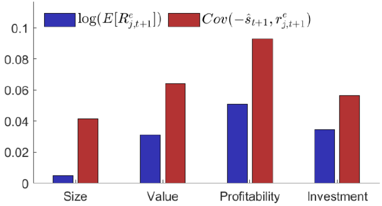

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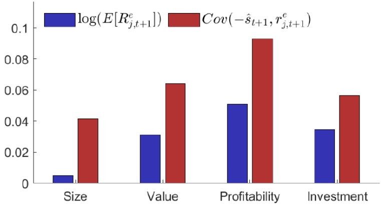

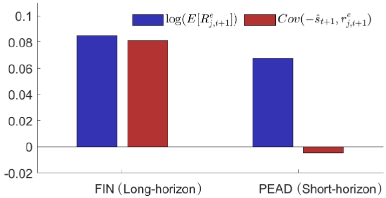

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PLEASE CLARIFY
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Thanks! We'll revisit them after the revision.
REVISING IS MY CARDIO
AUTHORS
R&R FIRST
When the paper already has an R&R

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- The paper uses the $\mathbb{E}_t^*[Z]$ equation to validate the SBF
- But we can use this equation as a belief imputation method
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  - $Z_{t+1} = \mathbb{E}_{j,t}^*[X]$ (study second order beliefs since $\mathbb{E}_t^*[Z] = \mathbb{E}_t^*[\mathbb{E}_{j,t}^*[X]]$)
  - $Z_{t+1} = \text{Asset Supply}_{t+1}$ (study how views about supply affect demand)
  - etc...

1) A Different use for the SBF: Belief Imputation

- This Paper: if X_{t+1} is a future outcome, coherent beliefs imply

$$\mathbb{F}_t[X] \;=\; \mathbb{E}^*_t[X] \;=\; \mathbb{E}_t[\mathrm{SBF}\cdot X]$$

  - Moreover, we can easily obtain the SBF that is linear in X:

$$\mathrm{SBF}_{t+1} \;=\; 1 \;+\; \lambda_t^{'}(X_{t+1} - \mathbb{E}_t[X]) \quad \text{where} \quad \lambda_t \;=\; \Sigma_t^{-1}\left(\mathbb{E}^*_t[X] - \mathbb{E}_t[X]\right)$$

  - We can then obtain $\boxed{\mathbb{E}^*_t[Z] = \mathbb{E}_t[Z] + \beta_t^{'}\left(\mathbb{E}^*_t[X] - \mathbb{E}_t[X]\right)}$

- The paper uses the $\mathbb{E}^*_t[Z]$ equation to validate the SBF
- But we can use this equation as a belief imputation method
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3) Using the BHF to Impute CMAs

$$\boxed{\mathbb{E}^*_{j,t}[Z] = \mathbb{E}^*_t[Z] + \beta^{*'}_t \left(\mathbb{E}^*_{j,t}[X] - \mathbb{E}^*_t[X]\right)} \quad \text{where} \quad \boxed{\beta^{*'}_t = \Sigma^{*-1}_t \mathbb{C}ov^*_t[X, Z]}$$

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FIXED INCOME	Compound Return 2022 (%)	Arithmetic Return 2022 (%)	Annualized Volatility (%)	U.S. Inflation	U.S. Cash	U.S. Intermediate Treasuries	U.S. Long Treasuries	TIPS	U.S. Aggregate Bonds	U.S. Securitized	U.S. Short Duration Government/Credit	U.S. Long Duration Government/Credit	U.S. Inv Grade Corporate Bonds	U.S. Long Corporate Bonds	U.S. High Yield Bonds	U.S. Leveraged Loans
U.S. Inflation	2.30	2.31	1.39	1.00												
U.S. Cash	1.30	1.30	0.42	0.05	1.00											
U.S. Intermediate Treasuries	2.10	2.14	2.81	-0.21	0.22	1.00										
U.S. Long Treasuries	1.80	2.44	11.55	-0.17	0.07	0.82	1.00									
TIPS	2.10	2.23	5.16	0.08	0.07	0.57	0.53	1.00								
U.S. Aggregate Bonds	2.60	2.66	3.48	-0.17	0.10	0.78	0.82	0.72	1.00							
U.S. Securitized	3.10	3.13	2.34	-0.17	0.15	0.75	0.68	0.64	0.89	1.00						
U.S. Short Duration Government/Credit	2.10	2.11	1.50	-0.19	0.35	0.77	0.49	0.58	0.76	0.71	1.00					
U.S. Long Duration Government/Credit	2.30	2.74	9.52	-0.17	0.01	0.66	0.88	0.63	0.93	0.72	0.55	1.00				
U.S. Inv Grade Corporate Bonds	2.80	3.00	6.38	-0.10	-0.01	0.33	0.46	0.66	0.77	0.58	0.51	0.77	1.00			
U.S. Long Corporate Bonds	2.40	2.93	10.47	-0.13	-0.03	0.34	0.56	0.60	0.79	0.57	0.43	0.87	0.96	1.00		
U.S. High Yield Bonds	3.90	4.22	8.24	0.10	-0.12	-0.28	-0.23	0.34	0.20	0.15	0.11	0.16	0.58	0.50	1.00	
U.S. Leveraged Loans	4.70	5.00	7.89	0.27	-0.15	-0.52	-0.41	0.12	-0.05	-0.09	-0.13	-0.06	0.37	0.29	0.81	1.0

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$$\mathbb{E}^*_{j,t}[Z] = \mathbb{E}^*_t[Z] + \beta^{*'}_t \left(\mathbb{E}^*_{j,t}[X] - \mathbb{E}^*_t[X]\right) \quad \text{where} \quad \beta^{*'}_t = \Sigma^{*-1}_t \mathbb{C}ov^*_t[X, Z]$$

IXED INCOME

	COMPOUND RETURN 2022 (%)	ARITHMETIC RETURN 2022 (%)	ANNUALIZED VOLATILITY (%)	U.S. Inflation	U.S. Cash	U.S. Intermediate Treasuries	U.S. Long Treasuries	TIPS	U.S. Aggregate Bonds	U.S. Securitized	U.S. Short Duration Government/Credit	U.S. Long Duration Government/Credit	U.S. Inv Grade Corporate Bonds	U.S. Long Corporate Bonds	U.S. High Yield Bonds	U.S. Leveraged Loans
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U.S. Short Duration Government/Credit	2.10	2.11	1.50	-0.19	0.35	0.77	0.49	0.58	0.76	0.71	1.00					
U.S. Long Duration Government/Credit	2.30	2.74	9.52	-0.17	0.01	0.66	0.88	0.63	0.93	0.72	0.55	1.00				
U.S. Inv Grade Corporate Bonds	2.80	3.00	6.38	-0.10	-0.01	0.33	0.46	0.66	0.77	0.58	0.51	0.77	1.00			
U.S. Long Corporate Bonds	2.40	2.93	10.47	-0.13	-0.03	0.34	0.56	0.60	0.79	0.57	0.43	0.87	0.96	1.00		
U.S. High Yield Bonds	3.90	4.22	8.24	0.10	-0.12	-0.28	-0.23	0.34	0.20	0.15	0.11	0.16	0.58	0.50	1.00	
U.S. Leveraged Loans	4.70	5.00	7.89	0.27	-0.15	-0.52	-0.41	0.12	-0.05	-0.09	-0.13	-0.06	0.37	0.29	0.81	1.0…

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U.S. Long Duration Government/Credit	2.30	2.74	9.52	-0.17	0.01	0.66	0.88	0.63	0.93	0.72	0.55	1.00				
U.S. Inv Grade Corporate Bonds	2.80	3.00	6.38	-0.10	-0.01	0.33	0.46	0.66	0.77	0.58	0.51	0.77	1.00			
U.S. Long Corporate Bonds	2.40	2.93	10.47	-0.13	-0.03	0.34	0.56	0.60	0.79	0.57	0.43	0.87	0.96	1.00		
U.S. High Yield Bonds	3.90	4.22	8.24	0.10	-0.12	-0.28	-0.23	0.34	0.20	0.15	0.11	0.16	0.58	0.50	1.00	
U.S. Leveraged Loans	4.70	5.00	7.89	0.27	-0.15	-0.52	-0.41	0.12	-0.05	-0.09	-0.13	-0.06	0.37	0.29	0.81	1.0

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  (Couts, Gonçalves, Loudis (2023), Couts et al. (2024), Andonov et al. (2025))

- In the latest one, we estimate

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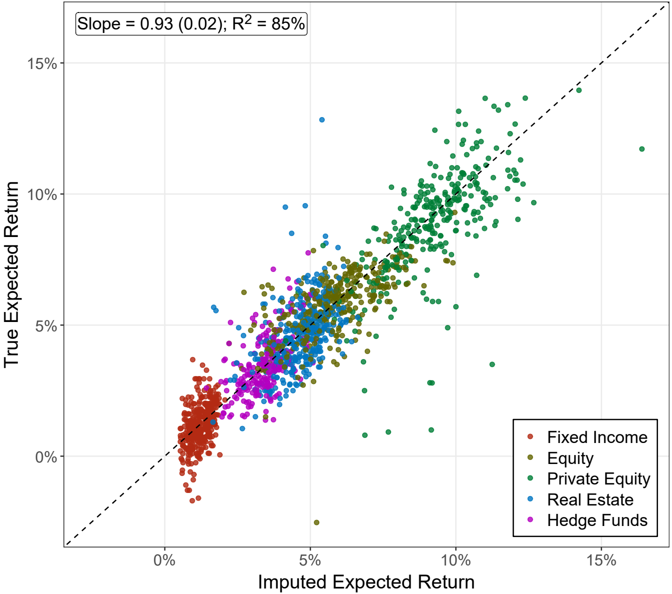

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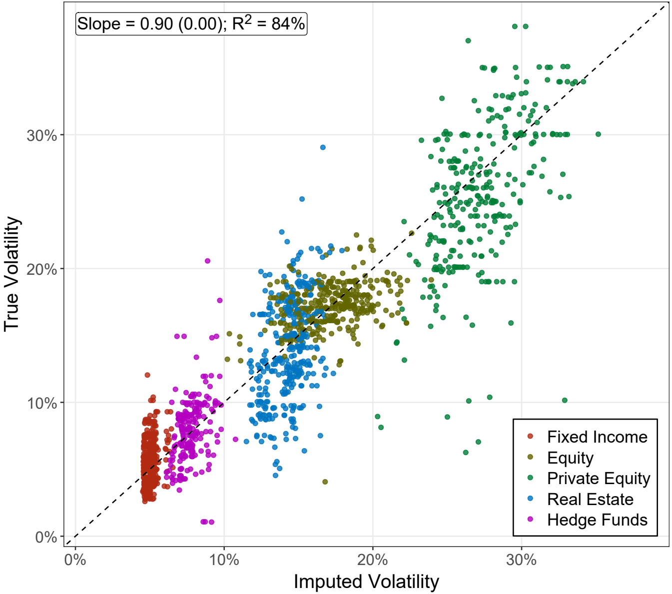

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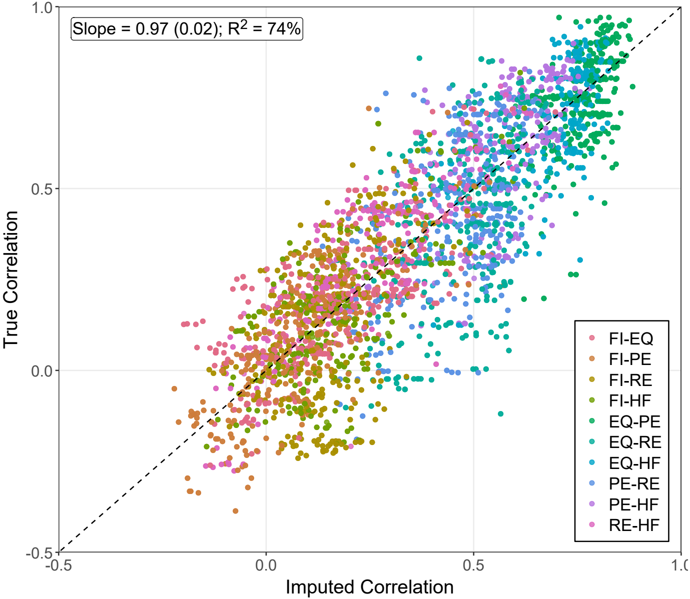

Outline

The Paper

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- This paper: $\mathbb{F}_t[X] = \mathbb{E}^*_t[X] = \mathbb{E}_t[\text{SBF} \cdot X]$
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But we still have
E[SDF·R]=1